Preference Modelling Approaches Based on Cumulative Functions Using Simulation with Applications

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Declaration

I declare that this thesis has not been, nor currently being, submitted for award of any other degree or similar qualifications.

Signed ____________________________________

Khwazbeen Saida Fatah
December 2009

I would like to dedicate my thesis in loving memory of my father to whom I am truly and eternally grateful

Acknowledgment

The completion of my work grants me the chance to sincerely express my deepest gratitude to everyone who has made a contribution and in any way helped me in achieving the end result. Praise is to God for bestowing upon me the passion and abilities as well as the opportunity to make this happen.

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Abstract

In decision making problems under uncertainty, Mean Variance Model (MVM) consistent with Expected Utility Theory (EUT) plays an important role in ranking preferences for various alternative options. Despite its wide use, this model is appropriate only when random variables representing the alternative options are normally distributed and the utility function to be maximized is quadratic; both are undesirable properties to be satisfied with actual applications.

In this research, a novel methodology has been adopted in developing generalized models that can reduce the deficiency of the existing models to solve large-scale decision problems, along with applications to real-world disputes. More specifically, for eliciting preferences for pairs of alternative options, two approaches are developed: one is based on Mean Variance Model (MVM), which is consistent with Expected Utility Theory (EUT), and the second is based on Analytic Hierarchy Processes (AHP). The main innovation in the first approach is in reformulating MVM to be based on cumulative functions using simulation. Two models under this approach are introduced: the first deals with ranking preferences for pairs of lotteries/options with non-negative outcomes only while the second, which is for risk modelling, is a risk-preference model that concerns normalized lotteries representing risk factors each is obtained from a multiplication decomposition of a lottery into its mean multiplied by a risk factor. Both approximation models, which are preference-based using the determined values for expected utility, have the potential to accommodate various distribution functions with different utility functions and capable of handling decision problems especially those encountered in financial economics. The study then reformulates the second approach, AHP; a new algorithm, using simulation, introduces an approximation method that restricts the level of inherent uncertainty to a certain limit.

The research further focuses on proposing an integrated preference-based AHP model introducing a novel approximation stepwise algorithm that combines the two modified approaches, namely MVM and AHP; it multiplies the determined value for expected utility, which results from implementing the modified MVM, by the one obtained from processing AHP to obtain an aggregated weight indicator. The new integrated weight scale represents an accurate and flexible tool that can be employed efficiently to solve decision making problems for possible scenarios that concern financial economics.

Finally, to illustrate how the integrated model can be used as a practical methodology to solve real life selection problems, this research explores the first empirical case study on Tender Selection Process (TSP) in Kurdistan Region (KR) of Iraq; it is considered as an inductive and a comprehensive investigation on TSP, which has received minimum consideration in the region, and regarded as a significant contribution to this research. The implementation of the proposed model to this case study shows that, for the evaluation of construction tenders, the integrated approach is an appropriate model, which can be easily modified to assume specific conditions of the proposed project. Using simulation, generated data allows creation of a feedback system that can be utilized for the evaluation of future projects in addition to its capability to make data handling easier and the evaluation process less complex and time consuming.

GLOSSARY OF TERMS

AHP	Analytic Hierarchical Process
CBA	Cost Benefit Analysis
CE	Certainty Equivalent
CI	Consistency Index
CR	Consistency Ratio
DPA	Data Protection Act
EU	Expected Utility
EUM	Expected Utility Model
EUT	Expected Utility Theory
IR	Inconsistency Ratio
KR	Kurdistan Region
MCDM	Multi-Criteria Decision Making
MVA	Mean-Variance Analysis
MVM	Mean-Variance Model
RI	Random Inconsistency Index
RU	Rank Uncertainty
TSP	Tender Selection Process
VNM	Von-Neumann and Morgenstern

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CHAPTER ONE: INTRODUCTION

The importance of decision-making problems under uncertainty has been widely recognized and different approaches in various fields such as operations research, management science, financial economics, and others, are developed to help decision-makers analyse their problems and weight alternative options.

Preferences guide human in making decisions and ranking them has become one of the methods to solve selection problems. However, ranking alternatives to select the most desirable one is often a critical problem especially when risk attitudes crucially affect these preferences in scenarios when huge losses and wins are possible; decision makers therefore need to take risk aspects into account to serve their users better. Therefore, the problem of ranking a set of alternatives arises in a number of ways and the theory concerning ranking has developed along the lines of the method of pair-wise comparison where only two alternatives are compared at a time; a choice between a pair of investment alternatives is an example.

The two approaches that have received broad applications in analyzing preferences in the field of economics are the applications of Expected Utility Theory (EUT), Von-Neumann and Morgenstern (VNM), (1944) and Mean-Variance Analysis (MVA), Markowitz (1952). In addition, Analytic Hierarchical Process (AHP), Saaty (1980), is a flexible model that provides a powerful tool and comprehensive framework for solving decision making problems to make the most preferable choice that maximizes profit with minimum risk, among a set of pre-specified competing alternatives that are evaluated under conflicting criteria. It deals with the qualitative variables and allows the decision makers to make decisions by personal judgments.

In the area of finance where investors are assumed to make their decisions among financial options, Mean-Variance Model (MVM), Markowitz (1952), has been used for portfolio selection based on minimizing variance subject to a given level of mean return. It is well known that this model is widely used; it is appropriate only when the random variables representing the alternative options are normally distributed or the utility function to be maximized is quadratic; both are undesirable properties to be satisfied with actual applications.

The objective of this research study is to investigate preference ranking techniques for solving decision making problems under uncertainty, especially decisions concerning business projects for financial investments. Based on reformulating MVM, relying on cumulative distribution function, to overcome the limitations of the existing models, as offered by the current literature, and provides solutions for the prevalent issues. For this purpose, using simulation, the study proposes a new preference ranking strategy, based on a new methodology, to solve large-scale pairs of financial scenarios. The new approach is then combined with AHP to establish a generalized AHP cost benefit model with a specified level of uncertainty, which results from the diversities in judgments; the new integrated approach is capable of handling not only quantitative but also qualitative, non-monetary, factors.

Therefore, the research focuses on a new modification for MVA, which is consistent with EUT, to be used for ranking pairs of lotteries or variables/uncertain alternative options with non-negative outcomes only. Based on cumulative distribution function, using simulation, it generates, from the inverses of the cumulative functions, different random values to be representative for such variables. The mean, the variance and then the expected utility for each of the generated random variables are determined, and then the preference ordering for each pair of such lotteries is determined and the more preferred one is identified. Hence, in order to incorporate risk attitudes into business decision problems, based on the same modification, a new modelling approach that links preference ordering of pairs of lotteries, with non-negative outcomes only, directly to a risk ordering, on risk factors, is introduced. Each risk factor, which is obtained from a multiplication decomposition of such lottery in to its mean multiplied by a risk factor, is defined as the ratio of the lottery relative to its mean and represented by a normalized random variable/lottery with the same expected value. With the existence of EUT, if the relative risk independence condition is satisfied then the preference ordering over any pair of such lotteries can be converted to a risk ordering on the risk factors obtained from such a decomposition structure, and then a risk-preference model, which can rank pairs of uncertain normalized lotteries/risk factors, is proposed.

Findings from the implementation of this modelling approach can ensure that regardless of their distributions, ranking alternatives are possible even if only variances are considered; this can overcome the limitation of other approaches (Sarin and Weber, 1993). Moreover, both models described under this approach, which are based on modifying MVM, are developed for analysing preferences for pairs of uncertain lotteries that represent financial scenarios for risk-averse investors; unlike the others, they can be applied to a variety of randomly distributed random variables with different utility functions.

The main shortcoming with the implementation of such ranking approaches is the way in which the preference information is processed; it is rather objective, which is based on the determined values for expected utilities; and the subjective judgments of decision makers have been ignored. Therefore, the study aims at establishing a new AHP-based approach, with a specified level of uncertainty, to propose an AHP cost benefit model that can deal with tangible in addition to intangible factors to allow decision makers to make decisions by personal judgments in a logical way with a pre-defined bound for uncertainty, which is resulted due to the diversity in judgments. Hence, the aim is to combine the two modelling approaches, the modified MVM and the new AHP, to conduct an overall method, an integrated AHP-based model, for eliciting preferences, using simulation results. With this approach, the preference information, which is obtained from applying MVM, is incorporated in to an AHP-based model to obtain a more efficient and integrated model that can be used as a flexible tool for ranking preferences. This new methodology provides a more accurate representation for weight indicators that easily reveal the strength and weakness of each alternative option; this enables the decision maker in explaining the reasons for selecting one rather than the other. Moreover, there is no need to impose any restrictions; it can be used to consider lotteries with various distribution functions or different utility functions.

However, a key motivation is to make this modelling procedure practical, to solve today's real-life problems; it is to be implemented in a field of application, construction industry is an example. Thus, an attempt is made to apply it as a systematic procedure for the evaluation of tenders for the process of tender selection in Kurdistan Region (KR) of Iraq. For this purpose, as a case study, through an investigation to the selection process, a comprehensive survey, based on the data extracted from a conducted pilot study for the same purpose is performed. The main objective of the survey, based on the expert's opinion in the region, is to identify the main criteria that are believed to have great impact on the selection process in addition to the main reasons that may lead to the failure of a project with the evaluation of their weights. Hence, through a questionnaire survey, various construction organizations, their type, size, classifications and other information in addition to the utilized tendering procedures with the criteria that has been based on in their decisions are investigated. Furthermore, the study reviews the criteria employed, by other countries, in the selection of the most qualified contractors and evaluation of tenders, then, it identifies, for KR, the most significant criteria that meet the circumstances and specific conditions, estimates their weights. The obtained data then can be included in the system of evaluation for selecting the most qualified contractor not only accords to the bid price but also according to the other quantitative and qualitative criteria.

Findings from this investigation reveal that tenders should not be selected according to the lowest price, but according to the highest weight determined from other qualitative criteria, such as past experience and past performance for the contractors before considering bid price; this supports the same findings for other countries. The survey evidences also show the influence of other significant criteria, the qualification of the tender and his staff; financial capability; resources on the selection process. Other criteria such as governmental support for construction organizations, private sector, should be considered in providing secure environment for the implementation of the tenders and the possibility of helping in making the materials prices settled. Moreover, it identifies, in addition to the lowest price, other specific reasons that lead to the failure of the projects in this region; due to high volatility, the currency fluctuation is believed to be the main reason that fluctuate material prices; this has great impact on implementing a successful project.

Finally, in order to bridge the theoretically developed models with the deductively constructed criteria to describe or solve today's real-life problems, to incorporate research evidence into practical act rather than being a theoretical concept, the thesis ends with the presentation of the model's implementation. The framework is applied to a field of applications, construction industry, and then the integrated model is used as a systematic procedure and applied to a field of financial investment scenarios in KR of Iraq, a specific case that has significant impact on the success of construction projects, the Tender Selection Process (TSP), with specific criteria, each with pre-determined weight, obtained from the main survey. Findings from this application, from the implementation of the integrated model, provide a more accurate representation and specific result for weight indicators that can be used as a basis for ranking preferences. With this integrated approach, in addition to the decision maker's judgments concerning the importance of the criteria; it includes information obtained from the implementation of other preference ranking approaches. Moreover, sensitivity analysis indicate that the evaluation of priorities are insensitive to the minor changes occurred in the judgments.

Furthermore, despite that the application of AHP, over the years, are only few in the area of construction, the proposed AHP-based model in this thesis is shown to be a practical and appropriate mean that can successfully be implemented to evaluate tenders for the process of tender selection in KR; it can easily be modified to assume specific conditions of the proposed project. Using simulation, generated data allow creating a feedback system that can be practiced for the evaluation of future projects in addition to its capability to make data handling easier and the evaluation process less complex and time consuming.

However, the study in this thesis, which introduces new methodologies and algorithms, contributes to research studies by proposing the first approximation approaches that are based on cumulative functions using simulation. Furthermore, the case study on TSP in KR of Iraq is regarded as the first comprehensive investigation on construction organizations for sectors, the public and private; it identifies the specific criteria that are significant for the selection of the most qualified tenders. Based on these criteria, it introduces the first systematic procedure to be implemented for the selection process in the region. Finally, this study contributes to the theoretical literature by two published papers and third submitted for further publication.

Following this introduction, in this chapter, the existing literature on decision making problems under uncertainty and their main concepts are reviewed.

1.1 Literature Overview

The importance of decision-making problems under uncertainty has been widely recognized (Baker and Shi, 2002; Zarghami and Szidarovszky, 2008), where every day each individual faces the problem of deciding which of a number of alternatives is most suitable or how to make more reliable and scientifically sound decisions. Different methods for various fields such as operations research, financial economics, decision or management science, and others have been developed to help decision-makers analyse their problems and weight alternative options (Brans and Vincke, 1985; Arbel and Vargas, 1993; Cook et al., 2005; Ellaz et al., 2007). Moreover, these studies have shown, despite that most people are less capable of decision making than they think, an understanding of what decision making involves, together with a few effective techniques, will help reach better outcomes.

In this chapter, an overview of the literature (Keeny 1982) on decision-making problems, under risk or uncertainty, decision analysis and its methodology are given, and then it identifies the problems and limitations of the rational decision-making model with an introduction to risk analysis and methods of risk analysis.

1.1.1 Decision-making problem and decision analysis

There has been much debate on how to define decision-making problems and their alternative options, and then how to choose the option that best fits with the main goals. There are significant differences between how people should make decisions and the way decisions are actually made. Despite that most people are rational beings, they reach decisions in accord with an underlying structure that enables them apply the law of probability to function predictably and systematically. However, decision making can be defined as the study of identifying and choosing alternative courses of actions, based on the values and preferences of the decision maker; most real world problems involve uncertain information, hence decisions can often be made based on incomplete knowledge or under uncertainty. Decision making under uncertainty has been addressed in mathematics by probability theory and expected utility theory. These two together are known as decision theory. The discipline that focuses on applying decision theory in practice is known as decision analysis; it offers a set of structured procedures that assist decision-makers in making decisions (Biswas, 1997).

Keeny (1982, P806) defines decision analysis as "a *formalization of common sense for decision problems which are too complex for informal use of common sense*", and gives a technical definition as "*a philosophy, articulated by a set of logical axioms, and a methodology and collection of systematic procedures, based upon those axioms, for responsibly analyzing the complexities inherent in decision problems*".

The foundations of decision analysis are provided by a set of axioms, which provide principles for analysing decision problems; stated first by Von-Neumann and Morgenstern (1944) then Savage (1954), Pratt et al. (1964) and later by Keeny (1982). In these axioms, all decisions require subjective judgments and the likelihood of different consequences and their preferences should be separately estimated using probabilities and utilities to calculate the expected utility of each alternative and the option with higher expected utility should be preferred more. This implies that these axioms include judgments and values, qualitative and quantitative factors, in the analysis of decision alternatives. Later in this chapter (Section 1.1.1.2) the set of axioms are overviewed.

1.1.1.1 The methodology of decision analysis

The methodology of decision analysis provides a framework to combine traditional techniques of operational research, management science and system analysis with professional judgments and values in a unified analysis to support decision-making problems.

Keeny (1982) decomposed this methodology in to four main steps:

1. Structure the decision problem which includes the generation of alternatives and the specifications of the objectives;
2. Assess possible impacts of each alternative; it is desirable to determine the set of possible outcomes, for each option, and the probability of occurring. This can be done formally by determining a probability distribution function over the set of outcomes for each alternative;
3. Determine preferences, values, of decision makers from an objective function, which is referred to as a utility function; the utility of a consequence indicates its desirability relative to all other consequences and alternatives, and higher expected utilities should be preferred to those with lower expected utilities;
4. Evaluate and compare alternatives, once a decision problem is structured, the magnitudes and the associated likelihoods of consequences are determined, and the preference structure is established. The information then must be synthesized in a logical manner to evaluate the alternatives; the basis for the evaluation of each option is the expected utility.

Thus, the main purpose of decision analysis is to help decision makers make better decisions and a key to successful decision analysis is the interaction of decision analysts, who try to formalise the thinking and feelings that the decision maker wishes to use on the problem, with the decision makers and other professionals working on the project. In the following section, a set of axioms, which provides principles for analysing decision problems, are introduced.

1.1.1.2 The axioms of decision analysis

The foundations of decision analysis are based on a set of axioms, which provide principles for analysing decision problems, stated first by Von-Neumann and Morgenstern (1944) then savage (1954), Pratt et al. (1964) and later by Keeny (1982). In these axioms, all decisions require perceptions or subjective judgments of the decision makers and the likelihood of different consequences and their preferences should be separately estimated using probabilities and utilities to calculate the expected utility of each option and the alternative with higher expected utility should be preferred more. This implies that these axioms include judgments and values, qualitative and quantitative factors, in the analysis of decision alternatives.

Keeny (1982, p830) states these as four main axioms; each concerns one of the main steps for the described decision analysis methodology, in the previous section (Section 1.1.1.1).

Axiom 1a (Generalization of Alternatives)*: At least two alternatives can be specified each one with a number of possible consequences which might result if that alternative option were followed.*

Axiom 1b (Identification of Consequences)*: possible consequences of each option can be identified.*

Axiom 2 (Quantification of Judgment)*: The relative likelihoods (i.e., probabilities) of each possible consequence that could result from each option can be specified.*

Axiom 3 (Quantification of Preference)*: The relative desirability (i.e. utility) for all the possible consequences of any option can be specified.*

Axiom 4a (Comparison of Alternatives)*: If two alternative options result in the same possible consequences, the alternative yielding the higher chance of the preferred consequence is preferred.*

Axiom 4b (Transitivity of Preferences)*: If one alternative is preferred to a second alternative and if the second alternative is preferred to a third alternative, then the first alternative is preferred to the third alternative.*

Axiom 4c (Substitution of Consequences)*: If an alternative is modified by replacing one of its consequences with a set of consequences and associated probabilities that is indifferent to the consequence being replaced, then the original and the modified alternatives should be indifferent.*

Thus, these axioms explain that making a decision is the degree to which the realities, in which decisions are made, deviate from the rational decision models of the Bernoulli's and VNM. The classical models of rationality, the models on which game theory and most of Markowitz's concepts are based, specify how people should make rational decisions in the face of risk. As a formal approach EUT proposed by VNM (1944), based on a set of logical axioms, emerged as a guiding framework of how to make rational decisions, according to this theory, a rational decision maker will maximize his or her utility by choosing the option with the highest utility. Therefore, the main result of these axioms is that the expected utility of an alternative is the indication of its desirability; alternatives with higher expected values should be preferred to those with lower expected utilities. This is explained in more detail in Chapter 2.

However, uncertainty is the key ingredient in many decision problems; financial investment is an example, in which ignoring uncertainty may lead to simply wrong decisions. Thus, it is important to define risk, uncertainty, and its role in risk analysis; it is explained in the following section.

1.1.2 Risk, uncertainty, and risk analysis

The concept of chance, risk, and uncertainty are as old as civilization. People have always had to cope with many risky aspects. As a result, they have devised risk management schemes that reduce uncertainty and improve the chance for their survival. Early risk management strategies were developed without formal logical structures. For example, games of chance have a long story, gambling with dice has been popular for many centuries. Gambling games were the motivation for the development of probability theory in the sixteenth and seventeenth centuries. In fact it is hard to consider any situation where risk does not play a role therefore it is important to define or to explain what do mean by risk or uncertainty, each term separately, then clarify whether there are differences in the meaning or both are equivalent.

1.1.2.1 Risk and uncertainty

Risk can be defined as representing any situation where some events are not known with certainty or a risky event to be defined as any event that is not known for sure ahead of time. Risk corresponds to events that can be associated with given probabilities and uncertainty corresponds to events for which probability assessment are not possible or difficult to be obtained. However, it is believed that there is no sharp distinction between risk and uncertainty, the probability would provide a tool (if not the best) for assessing, measuring and controlling risk (Chvas, 2004).

To understand the meaning of risky events and to clarify the consequences, it is important to know the main factors that contribute to their existence and prevalence. The main factors contributing to the existence of risk can be summarised as:

1- Inability to control or to measure precisely some significant factors of events;
2- Limited ability to obtain and to process information;

A unified theory that has attempted to put some structure on and represent risky events is probability theory.

However, people differ in how much they are willing to take risks; they show variety of risk attitudes while their judgment about risky events depends on both the probability and the magnitude of adverse effects. The choices that people might make to handle risky events could differ significantly; that is why some people are willing to buy lottery tickets, but at the same time insure themselves against theft, death, or property damages. Risk attitudes are explained in more details in the next chapter.

In the following section, in order to gain an understanding of a degree of risk involved, it would be helpful to examine risk and risk analysis; which are necessary components of an active life.

1.1.2.2 Risk analysis

In daily life individuals are surrounded by different kinds of risk and they constantly struggle to find better methods to quantify and manage them. There are different types and levels of risk that they are unwilling to accept. In risk analysis, one tries to recognize the nature of various kinds of risk and to assess their magnitude. During the past decades a few studies have been carried out on risk related topics and the field of risk analysis.

For example, Nsman, (2005, P6) defines risk as "*the interdisciplinary field of science that combines results and knowledge of probability theory, mathematical statistics, engineering, medicine, philosophy, psychology, economics and other applied disciplines*", and insures that in a risk analysis the following main steps should be followed:

1- Perception, it is the way we perceive the risks. It is associated with the psychological degree of risk and it is very important to understand how to interpret the risks;
2- Identification, the risk identification is to pose questions like what can happen or what can go wrong that could lead to negative consequences;

3- Estimation, is to estimate the risks and likelihood for the different events/or outcomes. Statistical methods with risk analysis methods can be used;
4- Valuation of risk; this step is an important one to evaluate risk and find possible ways of controlling and reducing it. The valuation of the risks is always tied together to risk perception;
5- Decision, after going through all the above steps of risk analysis, different decision options should be compared with respect to the results of the risk analysis together with the benefits and the costs as a basis for the decision.

Thus, risk analysis is the process of defining and analyzing dangers to individuals, businesses and agencies; it is generic and may be applied to any situation form of decision-making. In the context of business decision making where risk refers to the variability of expected returns, in which statistics such as variance and coefficient of variation are used to measure various risks, measuring and analyzing the associated risk is an important task; risk analysis is especially used in making capital investment decisions. However, for these decisions, where large amount of capital may involve in long-term investments, the higher the risk associated with a proposed project the greater the return that must be earned to compensate for that risk. Therefore, risk analysis may be used to develop an organizational risk profile, and also may be the first stage in a risk management program.

Chavas (2004) is one of the best sources to have given an overview for types of risk analysis, risk assessment and risk management.

1.1.2.3 Quantitative and qualitative risk analysis

Quantitative risk analysis attempts to transfer everything to monetary values. It assigns a dollar value not only to the risky prospects (such as assets) themselves, but also to the outcomes (threats and the harms they could produce). Making a decision about what to protect, and how much to spend protecting it, becomes an exercise in mathematics. The most important benefits or advantage of this technique is the use of familiar probability languages, which is mathematically based, and the use of a terminology, to derive and conduct results, familiar to management. However, the disadvantages of quantitative techniques are also significant. Some prospects are intrinsically not quantitative. How does one assign a dollar value to individual's morale, satisfaction, or public image? Any such assignment would have some amount of subjectivity to it, which is by definition qualitative, not quantitative. Furthermore, it sometimes generates a massive amount of data which requires some complex calculations.

While qualitative risk analysis relies strongly on people's opinion, experience, and intuition. It uses a variety of polling, interview, and questionnaire techniques to rank prospects by their perceived likelihood. This technique has its advantages; it is a simple and easily understood as long as the right people are expressing their opinion, it identifies significant risk areas. It can evaluate and conduct, then yields satisfactory results, but not in monetary terms. The most serious disadvantage for this technique is that it is difficult to enforce any degree of consistency and uniformity; people are asked to weight prospects in the light of terms like "extremely critical", "critical", "important", "low", which can vary from one person to another.

1.1.2.4 Risk assessment and risk management

The final stage of a risk analysis is the risk assessment. It is defined as structured methods for identifying, analyzing, and evaluating risks, which provides useful support and contribute significantly to decision making and regulatory process, it gives information but not solutions, the risk manager still has the task of deciding whether the assessed risk is acceptable or not, if so, based on information from a number of sources, what should be done?. Risk assessments are typically based on statistics and historical data, a quantitative risk assessment is computationally involved, but in cases that statistical data do not exist, or are not available, or are insufficient, an alternative is to obtain information form experts or qualitatively. Finally, risk management is the task of selecting an appropriate response to identified risks by measuring, or assessing risk and then developing strategies to manage the risk.

1.2 Aims and Objectives of the Study

In the area of finance where investors are assumed to make their decisions among financial options, Markowitz (1952; 1959; 1987) proposed MVM for portfolio selection based on minimizing variance subject to a given level of mean return. However, it is well known that this model is widely used, it is appropriate only when the random variables representing the alternative options are normally distributed or the utility function to be maximized is quadratic; both are undesirable properties that can limit the applicability of this model in real life applications.

The present study research is aiming to propose generalized models that can overcome the deficiency of MVM; it is to reformulate this model to be based on cumulative distribution functions, and then introduce a new methodology that proposes a new preference ranking strategy. Instead of only requiring normal random variables, representing alternative options/lotteries, with probability distribution functions $f(x)$; it uses random variables with different cumulative functions, which are increasing functions and possess inverses. It can easily generate, from the inverses of cumulative functions, cumulative random variables, to represent such alternative options, using simulation. Hence, comparison between these alternatives can be approached by examining the determined expected utilities for their associated cumulative variables.

However, for financial scenarios, the decision is mainly depended on the associated risk, which is measured by its associated variance; the main objective is to propose, using the same methodology but based on the variance, a risk-preference model for ranking lotteries/ random variables, which can be represented by normalized lotteries/ risk factor with the same expected values. Each risk factor, which is obtained from a multiplication decomposition of the random variable into its mean multiplied by a risk factor, is defined as the relative ratio of the random variable to its mean. The risk-preference ranking helps decision makers make their preference choices by comparing the determined values of the expected utilities, which are computed based on the determined values of variances, for the generated cumulative variables.

Meantime, another objective of this thesis is to include the intuitive perceptions, or judgments of decision makers/ knowledgeable individuals or experts, with a specified level of uncertainty that is measured by the average variance of the judgments, in to the decision process. Therefore, the study aims at proposing an AHP-based model, in which the suppliers of judgments for each pair-wise comparison matrices usually play an important role in specifying the hierarchy. In order to establish a common and appropriate model that enables decision makers make their decisions, based on various criteria, qualitative in addition to quantitative, the integration of an AHP with the proposed preference ranking models is another objective.

Finally, the main aim is to apply the proposed models to a real-world problem. Thus, for the evaluation of construction tenders in Kurdistan Region, to select the most qualified one, conducting an investigation is another objective. It is aimed at identifying the main criteria that have significant impact on the selection process. Then, in order to validate the proposed model, its implementation against experimental results is another objective; it is to be practised in realistic decision making scenarios.

1.3 Organization of the Thesis

This thesis deals with a new modification for MVA consistent with EUT and introduces a new ranking strategy, namely, preference-based models that can rank pairs of alternatives options; risk-preference models for ranking pairs of normalized lotteries with the same expected values. It is based on two working published papers on the topic of preference ranking models based on both MVA and cumulative function, using simulation (Fatah, et al., 2009a; 2009b). Hence, it proposes an AHP-based model with specified level of uncertainty;

and finally an integrated AHP with preference ranking models for ranking pairs of alternative options, each only with non-negative outcomes, along with an application to a real-world life, financial investment, scenario; it is based on a working paper on the topic of Construction Engineering Tender Selection: A Case Study in Kurdistan Region of Iraq (Fatah, et al., 2009c), which is submitted for further publication. In this view, the thesis is organized in as follows:

Following this introduction, in order to modify both MVM, which is consistent with EUT, and AHP to develop a new approach for analysing preferences for pairs of uncertain alternative options, in Chapter 2, methodology overview for the main theories and background information on which this thesis is built, are reviewed. The basic concepts for EUT and its main axioms and main results, which are used to introduce the simplest investment model for financial economics; namely the MVM, are provided. Hence, in the context of EUT, risk attitudes are explained. Later in this chapter an AHP model and its main concepts and hierarchy construction with setting up judgmental matrices are introduced.

Based on reformulating MVM and relying on cumulative function, using simulation, Chapter 3 introduces a new methodology that proposes an approximation model, which can overcome the deficiency of the existing MVM. Using simulation, it generates various random values from the inverses of cumulative function to represent the alternative options. Based on the determined expected utilities, this approximation approach can rank pairs of generated random variables, with non-negative outcomes only. This modelling procedure has many desirable properties; it is a general ranking technique based on simulation results that can be applied to lotteries with different probability distribution function and various utility functions; it can be applied to solve decision making problems for financial scenarios.

When risk is an important component and its measure is independent of the expected value for a lottery, Chapter 4 introduces a powerful ranking strategy for risk judgments based on normalized lotteries with the same expected values. Each one is obtained as the result of multiplication decomposition for a random variable into its mean and a risk factor/normalized lottery. With the existence of EUT, it converts the preference ordering over any pair of such lotteries into risk ordering on risk measures, which is obtained from multiplication decomposition; hence, based on cumulative function, using simulation, a risk-preference model is proposed. Regardless of their distributions, this risk-preference strategy can be applied to scenarios/lotteries, which are with different utility functions, when huge loses and gains are included, application in financial investments. As there is no need to consider the determination of means, the preference ordering would then be determined by just focusing on their risk measures.

In order to overcome the shortcoming of the proposed models in Chapters 3; 4, in which the preference information is processed based only on the determined values of expected utilities; Chapter 5 introduces an AHP-based model which includes judgments of knowledgeable experts/decision makers into the decision problems. To restrict the uncertainty, which is resulted from the judgmental diversities for the experts and measured by variances, to a desired limit, the first section of this chapter proposes an algorithm. Using simulation, the algorithm generates a preference matrix with an average variance that does not exceed the desired level of uncertainty. To demonstrate the algorithm, it is applied, as an example, to a scenario with a defined preference matrix; it is run for different values of uncertainty limit. Hence, to determine an aggregated measure for uncertainty, the algorithm computes the Rank Uncertainty for the generated Eigen-vectors. The second section of this chapter describes a new AHP-based model, which is with a specified level of uncertainty, combined with the preference ranking models to propose an integrated AHP preference-based model. The integrated measurement obtained by this combination, which is based on more than one weight scale, can provide a powerful measure scale that can be used as a basis indicator for ranking pairs of options. There is no need to impose restrictions; it can solve large scale problems for financial scenarios such as financial investment.

To demonstrate the applicability of the proposed approaches, Chapters 3; 4; 5, to incorporate research evidence into practical act rather than being a theoretical concept, Chapter 6 applies the framework to a field of applications, construction industry, the Tender Selection Process in Kurdistan Region of Iraq. It explores an empirical case study on TSP, which is an inductive investigation on TSP that has received a minimum consideration in this region. By conducting a main survey, it reviews the criteria employed in the selection of the most qualified contractors and evaluation of tenders; based on experts knowledge, it identifies the most significant criteria to be included in the systematic procedure that is proposed to be used in the selection process in the region. Results of the case study, the simulated results, show the applicability of the proposed AHP-based model as a systematic procedure for evaluating tenders, which can be used as a basis for tender selection process in this region.

Chapter 7 is concerned with a conclusion on the main outcomes and an assessment of the objectives and the extent to which they have been fulfilled. Finally, some recommendations for future work are presented.

1.4 Summary

In this chapter, an introduction to the main topics, which are related to the main study in this thesis, is provided. As the study deals with preference ranking approaches for solving large-scale decision making problems under uncertainty, especially problems concern financial economics that involve huge gains and losses; this chapter, through literature review, defines decision-making problem and its main concepts; decision analysis and its methodology with the main axioms of decision analysis and decision theory. Uncertainty plays an important role in decision making problems that are studied; an introduction to risk and uncertainty along with risk analysis is provided. Later in this chapter the aims and objectives for this study are explained. Finally, the structure organization of the thesis is provided.

CHAPTER TWO: METHODOLOGY OVERVIEW

This thesis introduces new modifications for each of the Mean Variance Analysis (MVA) consistent with Expected Utility Theory (EUT) and the Analytic Hierarchy Process (AHP) to propose a new ranking strategy, which is based on these modifications, for ranking pairs of alternative options with non-negative outcomes only. This chapter, Methodology Overview, reviews theories and background information for some particular techniques on which this thesis is built. In section 2.1, it focuses on EUT and its main concepts including the principle of maximization of Expected Utility (EU) and EUT axioms; these are important tasks for building a Mean Variance Model (MVM). In section 2.2, Mean Variance Analysis (MVA) and its related concepts are studied and following this, the consistency with EUT is explained. The Analytic Hierarchy Process (AHP) is introduced in section 2.3, together with a study of its related topics.

2.1 The Expected Utility Theory (EUT)

The EUT, proposed in the 17th century by Cramer and then Bernoulli; later axiomatized by VNM (1944), has long been the predominant framework for the analysis of decision-making problems under risk. This theory is concerned with people's choices and decisions; it is designed to provide guidance on how to choose between alternative courses of action under conditions of uncertainty. For EUT, several systems of axioms, which all produce the VNM model, have been developed. The theory however, based on a set of axioms, considered as necessary for rational decision making, which studies individual's preference structures and their numerical representation in the form of utility theory. The assumptions of utility theory are usually stated in terms of an individual's preference relation, a binary relation $\succ$ ("is preferred to"), applied to X (the set of elements or outcomes $x, y, z, ..$), usually interpreted as decision alternatives or courses of action.

To discuss the axioms of EUT, it is needed to explain the technical term lottery. It can be defined as follows:

Definition 2.1: Consider n possible outcomes in an outcome space $A = (A_1, A_2, .., A_n)$, a lottery L is the set of all possible outcomes in A with an associated probability density function defined over them. Then the lottery L can be expressed as $L = [p_1 : A_1; p_2 : A_2; ...; p_n : A_n]$ where $A_1, A_2, .., A_n$ are outcomes with associated probabilities $p_1, p_2, .., p_n$ respectively.

Remark 2.1

The following are the notation used with the lotteries within the EUT framework

$\succ$ Denotes preferable
$\sim$ Denotes indifference (equally preferable)

To describe the mathematical model for EUT, it is needed first to introduce the concept of utility in the context of this theory. For example, utility, in economics and finance, is described as a personal measure of satisfaction; it is the monetary value that is assigned to a specific goods, service, situation or action. This measure is supposed to be the main parameter which allows comparison with others so as to make a decision.

In the context of EUT, utility and utility function can be defined by:

Definition 2.2: A utility is a numerical rating assigned to every possible consequence or outcome a decision maker may be faced with, the rating must be such that the utility of any uncertain lottery is equal to the expected value, the mathematical expectation, of the utilities of all its possible outcomes.

Definition 2.3: A mathematical function that can represent utilities for all outcomes of a lottery is called a utility function and denoted by $U(.)$.

2.1.1 Mathematical model for EUT

The Expected Utility Model (EUM) proposed by EUT, the predominant approach for modelling risky decision-making, has been the focus of much theoretical and empirical researches, including various interpretations and descriptive modifications as to its mathematical form (Machina, 1982; Rabin and Thaler, 2001). The model is concerned with choices among risky options whose outcomes may be either single or multi-dimensional. The mathematical form of this model stems back to the 17th century during the development of modern probability theory. Mathematicians Blaise Pascal and Pierre de Fermat assumed that individuals would evaluate alternative monetary gambles on the basis of their expected values, so that the lottery, represented by a random variable X, offering the payoffs $(x_1, x_2, .., x_n)$ with respective probabilities $(p_1, p_2, .., p_n)$ would yield as much satisfaction as a sure payment equal to its expected value $\bar{x} = \sum_1^n x_i p_i$. This implies that decision-making maximizes the expected return that provides a simple model of decision-making under uncertainty. This means that people behave in a way consistent with the maximization of expected values.

However, the fact that individuals can evaluate more than expected value was illustrated by Bernoulli (1738), which was translated by Sommer (1954), in his discussion of decision-making under uncertainty, in response to a broad puzzle that concerns a rational price a person should be prepared to pay to enter a gamble. It was reasonable, at that time, to pay anything up to the expected value of the gamble, but Bernoulli introduced his counter example which is known today as the St. Petersburg Paradox (Biswas, 1997). Bernoulli presented a solution to this paradox, specifically; he outlined sensible reasons why individuals would pay only a small amount for a game with an infinite mathematical expectation. He disputed that a $\$100$ gain was not necessarily worth twice as much as a gain of $\$50$, suggesting that individuals maximize expected utility $\overline{u} = \sum u(x_i)p_i$ rather than expected monetary value $\overline{x}$, and a defined expected utility model was one that maximizes the expected utility. The utility function $U(x)$ he proposed was logarithmic, which presented diminishing increases in utility for equal increments in wealth and did, in fact, show that the expected utility is indeed finite in such cases. Two centuries later, this approach was formally axiomatized by VNM (1944) and others.

Thus, the Axioms of EUT as provided by VNM (1944) are summarised below:

2.1.2 Axioms of EUT (Axioms of Rational Choice)

- Completeness: A decision maker (or an agent) must be able to state her/his preference for all outcomes (alternatives) of a lottery. i.e. given any two outcomes, A and B, a decision maker prefers one of them, else the two are equally preferred, $(A \succ B) \vee (B \succ A) \vee (A \sim \mathrm{B})$;
- Transitivity: Given any three outcomes A, B, C, if a decision maker prefers A to B and prefers B to C, then he must prefer A to C, $(A \succ B) \wedge (B \succ C) \Rightarrow (A \succ C)$;
- Continuity: For any three outcomes A, B, C, for which $A \succ B \succ C$ there exist a scalar p, $0 < p < 1$, for which the rational decision maker will be indifferent between B and the lottery in which A comes with probability p, C with probability $(1-p)$, $(A \succ B \succ C) \Rightarrow \exists p : B \sim [p : A; (1-p) : C]$;
- Independency: Given any three outcomes A, B, C, a decision maker prefers A to B independent of C (the preferences between A, B are independent of C), $(A \succ B) \Leftrightarrow [p : A; (1-p) : C] \succ [p : B; (1\text{-}p) : C]$ for any value of p.

Therefore, if a decision maker accepts the above axioms as valid then an expected utility function exists such that the decision maker's preferences for lotteries will correspond with the utility values generated by the utility function. Hence, a fundamental result from utility theory is the existence of utility function that yields the following:

2.1.3 The main results for EUT

If the axioms of EUT are followed then the following main results could be obtained:

1. There exist a real valued function $U(.)$ such that
   - $U(A) > U(B) \Leftrightarrow A \succ B$
   - $U(A) = U(B) \Leftrightarrow A \sim B$

   Where A and B are any two outcomes for a lottery and $U(.)$ is the utility function.
2. The expected utility of the lottery with outcomes A, B is the sum of utilities of outcomes weighted by their probabilities; i.e.
   $E[U(p : A; (1-p) : B)] = p * U(A) + (1-p) * U(B)$
3. Rational decision makers make their decisions for a pair of lotteries in the presence of uncertainty by maximizing its expected utility, i.e. for any pair (L_1, L_2) of lotteries $L_1 \succ L_2 \Leftrightarrow E[U(L_1)] > E[U(L_2)]$

Remark 2.2: EUT explains behaviour of individuals in terms of preferences over probability distributions and introduces a formal model of decision-making under risk namely: the EUM, which assumes that each decision-maker has a utility function representing his risk preferences. This explains that, facing uncertainty; people behave or should behave as if they were maximizing the expectation of some utility function of the possible outcomes. If the probability distribution function are not given then people should behave as if they were maximizing some utility function relative to some probability measure. In the context of risk, namely, with known probabilities, it provides a definition of the concept of "utility". This means that a formal relationship between the properties of the utility function and risk preferences can be established and this will provide some useful insights into the empirical analysis of risk behaviour.

2.1.4 Mathematical preliminaries

Chavas (2004) presents some mathematical concepts that will prove useful results in the analysis of risk behaviour. A key concept is the concavity (or convexity) of a function. A function $U(x)$ is said to be a concave function, if for any α, $0 < \alpha < 1$, and x between any two points a_1 and a_2, $U(\alpha a_1 + (1-\alpha)a_2) \geq \alpha U(a_1) + (1-\alpha)U(a_2)$

And $U(x)$ is a convex function, if for any α, $0 < \alpha < 1$, and any two points a_1 and a_2

$U(\alpha a_1 + (1-\alpha)a_2) \leq \alpha U(a_1) + (1-\alpha)U(a_2)$

If it is known that the function $U(x)$ is twice continuously differentiable, then

- $U(x)$ is concave if and only if $\partial^2 U / \partial x^2 \leq 0$ for all x
- $U(x)$ is convex if and only if $\partial^2 U / \partial x^2 \geq 0$ for all x

And the important property of concave (convex) functions is stated as Jensen's inequalities:

If $U(x)$ is a $\begin{Bmatrix} concave \\ linear \\ convex \end{Bmatrix}$ function of a random variable X then

$$U(E[x]) \begin{Bmatrix} \leq \\ \equiv \\ \geq \end{Bmatrix} E[U(x)]\text{, where } E \text{ is the expectation operator.}$$

Therefore, in the context of EUT, risk attitudes can entirely be captured by the curvature of the utility function. In this framework, risk attitudes can take different forms; risk aversion is the most important behaviour.

2.1.5 Risk aversion

Risk-averse means being willing to pay money to avoid risk or, when faced with two courses of actions, options, with a similar expected return, but different risks, will prefer the one with the lower risk. In the sense of Bernoulli utility function, if the utility of the expected value of a gamble, lottery, is greater than his expected utility from the gamble itself, the decision maker is said to be risk-averse. This is a more precise definition of Bernoulli's idea. Risk-averse behaviour is captured by a concave Bernoulli utility function, like a logarithmic function. For example, a person, whose Bernoulli utility function is logarithmic and takes the form $u(w) = \log(w)$, is risk-averse. The concept of risk aversion is linked with the idea of a fair bet; a fair bet is uncertain prospect or lottery whose expected value is zero. A person is risk averse if he never accepts a fair bet, this is equivalent to the familiar assumption that his utility function is concave.

In another case when a person's utility of the expected value of a gamble is less than his expected utility from the gamble itself, he is said to be risk-loving or risk lover, which means that he always accepts a fair bet, with a convex utility function. A convex Bernoulli utility function defined as exponential function.

Finally, if a person is always indifferent between accepting a fair bet and rejecting it, he is called a risk-neutral with a linear utility function. A risk-neutral person whose Bernoulli utility function takes the form of $u(w) = 2w$.

Remark 2.3

In most of the traditional literature dealing with risk and uncertainty, there is empirical evidence that most decision makers, economic agents, are assumed to be risk averse and the utility functions are concave.

Thus, risk aversion behaviour plays an important role in analysing preferences; two cases of risk attitudes can be identified. Next section explains both.

2.1.5.1 Absolute risk aversion

In the case of risk aversion, where $U' > 0$, risk aversion imposes a restriction on the sign of the second derivative of the utility function: $U'' < 0$. This means that a risk-averse individual has risk behaviour represented by a utility function that shows decreasing marginal utility with respect to his income. If $r = -U''/U' = -\partial \ln(U')/\partial w$, then integrating over r yields $\int r dw = -\ln(U') + c$ or $U' = e^c e^{-\int r dw}$, where c is a constant of integration. This implies that $U(w) = e^c \int e^{-\int r dw} dw + k$, where k is another constant of integration. Since $U(w)$ is defined up to a positive linear transformation, we can always choose $k = 0$ and $c = 0$. It follows that the utility function $U(w)$ can always be expressed exactly as $U(w) = \int e^{-\int r dw} dw$.

This case is called absolute risk aversion where r is the Arrow-Pratt coefficient of absolute risk aversion. This gives a hint that the properties of the Arrow-Pratt coefficient of absolute risk aversion r will provide useful information on the nature of risk preferences.

2.1.5.2 Constant absolute risk aversion

A special class of risk preferences is associated with the case where for the absolute risk aversion case $r = -U''/U'$ is constant. It is shown that the utility function can always be written as $U(w) = \int e^{-\int r dw} dw$, given that $U' > 0$ under constant absolute risk aversion, this gives the following

- $r > 0$ (risk aversion) corresponds to the utility function $U = -e^{-r*(x+a)}$
- $r = 0$ (risk neutral) corresponds to the utility function $U = x + a$
- $r < 0$ (risk loving) corresponds to the utility function $U = e^{-r*(x+a)}$

Where a is a constant; it is called initial wealth.

This shows that when $r = 0$, the utility function is linear, it also identifies the special importance of the exponential utility functions in risk analysis and their close linkages with constant absolute risk aversion. If $r \neq 0$ then $e^{-r*(x+a)} = e^{-rx}e^{-ra}$, this implies that the expected utility is proportional to $E[e^{-ra}]$ for any a. This means that changing the value for a does not affect economic decisions for all cases (i.e. whether $r = 0$, $r > 0$, or $r < 0$). Therefore, this result applies whether the decision maker is risk averse, risk neutral, or risk loving and shows that, under the expected utility model, an exponential utility function implies the absence of the effect of the value of a.

Remark 2.4

Another important property for a risk-averse decision maker, when the case is constant absolute risk aversion, if the random variable X is normally distributed with mean μ and variance V, and the utility function $U = -e^{-r*(x+a)}$, $r > 0$ then maximizing $E[U(.)]$ is equivalent to maximizing $[\mu - (r/2)V]$. This is an important property for empirical risk analysis (Chavas, 2004).

Based on the concepts of EUT, next section introduces a ranking preference approach that is consistent with EUT; namely Mean Variance Analysis (MVA).

2.2 Mean Variance Analysis (MVA)

In the previous section, the EUT, the formal theory of decision-making under risk, and the EUM, the dominant approach for modelling risky decision-making, were introduced. In this section, analyzing behaviour under risk in a mean variance context, under the EUM, is explained and the mean variance approach, given that the estimation of the first two moments of the distribution function is often relatively easy to obtain empirically, is introduced. Using MVA, individuals can choose alternative combinations that provide the maximum expected return for a given level of risk or, alternatively, the minimum level of risk for a given expected return.

2.2.1 Mean variance model (MVM)

The Mean Variance Model (MVM), introduced by Markowitz (1952), is the simplest investment model for decision-making problems for financial economics. With this model, the decision maker is assumed to rank alternative options according to the value of some function $f(\mu, V)$ defined over the first two moments the mean μ and the variance V. Given that the estimation of the first two moments of the distribution of a random variable X is often relatively easy to obtain empirically, this model has become an attractive one in applied risk analysis. Thus, it has been utilized in a variety of fields with the best developed, both theoretically, and empirically, is the portfolio selection.

2.2.2 Mean variance analysis consistent with expected utility model

The basic assumption for MVM is that individuals prefer higher returns and seek to avoid risk or variability of the returns where risk is measured by variance and returns by mean, and that the decision criterion should be to minimize variance given expected return, or to maximize expected return for a given variance. Therefore, analyzing behaviour under risk in a mean variance context implies that expected utility can be expressed as a function of mean and variance. If we consider a utility function $U(x)$ for an alternative represented by X, then the expected utility can be expressed as $E[U(x)] = f(\mu, V)$, where $\mu = E(x)$ is the mean, and V is the variance for X, this implies that both models, EUM and MVM, are consistent.

However, the two models, The EUM proposed by VNM (1944) and the MVM proposed by Markowitz (1952), have generated a considerable literature. They have been studied by many researchers, as some authors are concerned with the advantage and disadvantage for each model, while others are concerned with the consistency between the two models by specifying the conditions under which both yields at least approximately the same results (Samuelson, 1970; Meyer, 1987).

Although, the two different models, representing preferences over different alternative options, have been criticized by many researchers since the early 60's, they are in wide use and they remained the most powerful models in decision making under uncertainty. It was well known that if consistency between these two models was to be insured then some restriction must be placed on either the alternative preferences or the set of random variables comprising the choice set. All these restrictions are presented in the literature, such as requiring that the utility function for that alternative be quadratic or that the random alternatives be normally distributed. However, the Taylor series approximation can guarantee the consistency between the EUM and MVM, which is explored further in the next section.

2.2.3 Taylor-Series approximation to expected utility model and mean-variance model

Despite the evidence and studies shown that Taylor-series expansion may not be appropriate when the series does not converge; for example (Loistl, 1976), or the utility function is a non-polynomial; for example (Broch, 1969). Tsiang (1972) showed that non-polynomials can generally be expanded in to Taylor's series provided that they are continuous and have derivatives. Levy and Markowitz (1979) show that MVA can be regarded as a second order Taylor series approximation of standard utility function (such as power and the exponential utility functions). Hlawitschka (1994) extends the results of Levy and Markowitz and presents empirical evidence that when Taylor series converges or diverges, two-moment expansions may provide excellent approximations to expected utility. This research showed that:

1. When a Taylor series of expected utility diverges, then the truncated Taylor series, particularly second order expansion, may provide excellent approximation to expected utility;
2. When Taylor series does converge, adding more terms may worsen the approximation.

Furthermore, the usefulness of Taylor series approximation is strictly an empirical issue unrelated to the convergence of the infinite series.

Thus, given a utility function $U(X)$ for a random variable X, which can be written as the sum of its mean plus the deviation from the mean h ($h = X - \overline{X}$), i.e. $U(X) = U(\overline{X} + h)$, then the resulting Taylor series approximation of the utility function for X around the mean gives the following model

$$E[U(X)] = U(\overline{X}) + \frac{U''}{2} S^2 \tag{2.1}$$

Where, $\overline{X}$ is the mean and S^2 is the variance for X, $U'' < 0$ (Pratt, 1964), is the second derivative for the utility function around $\overline{X}$. Thus, from equation (2.1), the expected utility for a random variable can be analysed in terms of its two parameters, the mean and the variance.

Hence, results obtained by equation (2.1) make it possible to propose new preference ranking models based on both MVA, by considering the consistency with EUT, and cumulative distribution function using simulation; this will be explained in the next chapter.

Remark 2.5

Relying on cumulative functions and results from equation (2.1) help in reformulating MVM, to propose new approaches for analysing preferences, using simulation. This leads to establish a new preference ranking strategy, which overcomes limitations of the existing models, possible, along with applications to real world problems.

However, in decision making problems, especially decisions concern business projects for financial investments, evaluation of any new decision model, where an analytic way to reach the best decision is more preferable, is always necessary. There are various decision making techniques among which the Analytic Hierarchy Process (AHP) has been found as a useful and simple method to deal with. Next section gives an overview to this approach.

2.3 The Analytical Hierarchy Process Approach

One of the main difficulties in applying decision making approaches such as Cost Benefit Analysis (CBA), which evaluates the costs and benefits for each alternative option in the same scale based on monetary units, especially to decisions concern financial economics, ranking financial options, is that when it deals with a set of decision alternatives (actions) and a family of inter-related criteria and sub criteria, factors. However, when the decision process involves considerations of non-monetary factors that cannot be easily quantified, it is a case of Multi-Criteria Decision Making (MCDM) problem.

Since MCDM problems are studied, numerous approaches and techniques have been proposed. The Analytic Hierarchy Process (AHP), proposed by Saaty (1980), one of the more sophisticated approaches of multi-criteria analysis, has been seen as a possible alternative approach to comply with such criticism. In a decision making process, AHP (Saaty, 1980; 1986; 1990; 1994; 2001; 2003) is capable of handling not only quantitative but also qualitative criteria; it offers decision makers the possibility of assigning different values to non-monetary factors and helps to building a cost benefit model which uses tangible and intangible costs and benefits for financial values.

The AHP has been successfully applied in various decision making problems; it is more than just a procedure for selection problems, it provides a methodology for modelling unstructured problems in economics, social, and management science (Saaty, 1986). It is a framework of logic and problem solving that organizes perceptions, feelings, judgments and memories into a hierarchy of forces that effect decision results, it is based on inherent human ability to use information and experience to estimate relative scales on a variety of dimensions, both tangible and intangible, through paired comparisons. Finally, it, arrange these dimensions in a hierarchic structure that allows a systematic procedure to organize all believes and intuitions by breaking down, the problem under consideration, into smaller parts. A typical hierarchy includes three main levels:

- The first level is the goal or focus level which specifies the overall objectives of the decision problem;
- The second level is criteria and sub-criteria level, it identifies all important criteria, sub-criteria which can be identified as sub-levels. For complex cases, there may exist several sub-criteria levels;
- The alternative level identifies all possible alternatives.

Briefly, the AHP method is based on three main functions or principles: (1) structuring complexity, by identifying all elements (criteria, sub-criteria or alternative) and then mapping their interrelations (2) measuring ratio scales, by determining the relative weights of the elements (3) synthesizing, aggregating their total effect, weight, in regard with each single criterion. These principles will be introduced in more detail after reviewing the axiomatic foundation of the AHP.

2.3.1 The original AHP method and its foundation

Since its development (Saaty, 1980) more than two decades, AHP has become a widely popular technique in decision making problems (Zahedi, 1986; Lin et al, 2008). The foundation of the AHP is a set of axioms based on the well-defined mathematical structure of consistent matrices and their associated Eigen-vectors to generate approximated weights. The AHP methodology compares criteria, or alternatives with respect to a criterion, in a pair-wise comparison procedure, using a fundamental scale of absolute numbers, 1-9, that has been proven in practice and validated by decision problem experiments. It derives ratio scales in the same way as in dealing with the objects and phenomena of the physical world. The fundamental scale has been shown to be a scale that captures individual preferences with respect to quantitative and qualitative attributes better than other scales. (Saaty, 1980;1994; Lin et al, 2008).

Thus, AHP is aimed at integrating different measures into an overall score for ranking all alternatives involved in the decision process. It allows structuring complexity and exercising judgment and allowing them to include both objective and subjective considerations in the decision process. It organizes the basic rationality by breaking down a problem in to its smaller parts and then calls for only simple pair-wise comparison judgments to develop priorities in each hierarchy.

However, the AHP contains an inherent measure called the Inconsistency Ratio (IR) which is used to test the consistency of judgment that will occur when the decision maker is not accurate in his judgment during the process of pair-wise comparisons; the degree of consistency (or inconsistency) of the judgments is revealed at the end of the process. Later, in this section, the consistency within AHP and the way of measuring it are explained. Furthermore, AHP is a compensatory decision methodology because alternatives that are insufficient with respect to one or more factors (objectives) can be compensated by their performance with respect to other factors.

2.3.2 Principles of the Analytic Hierarchy Process

In decision making problems using AHP, Saaty (1986) proposed three main principles; they are the principles of identity and decomposition, the comparative judgments, and synthesis of priorities.

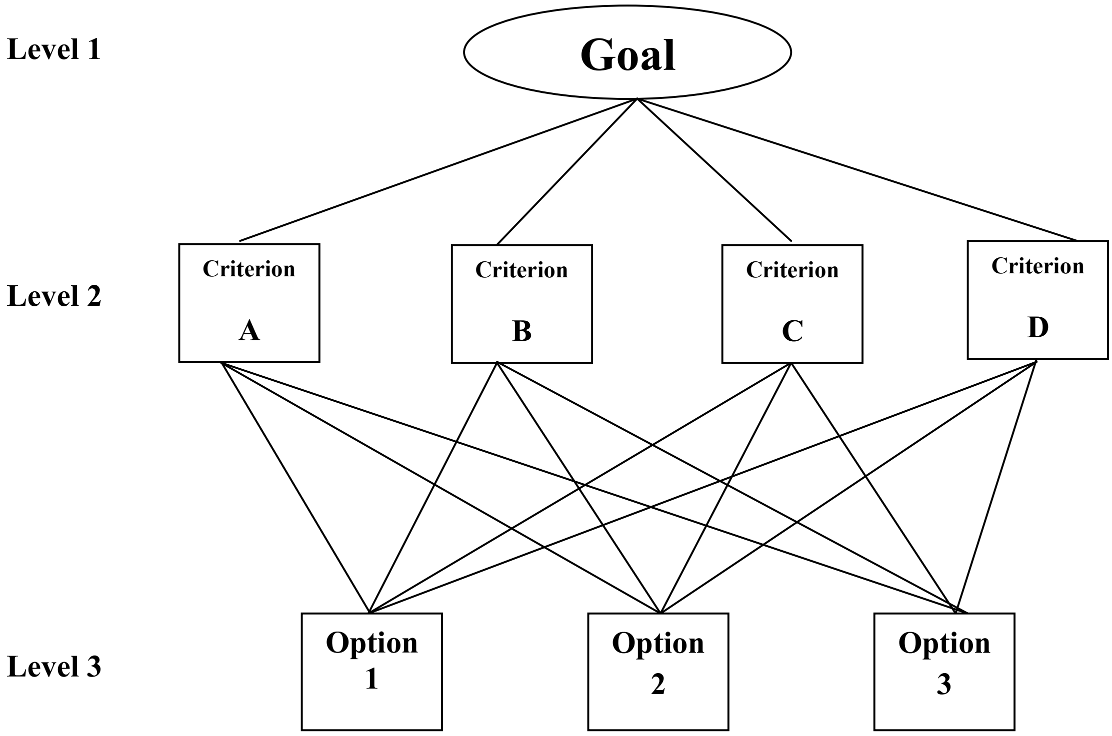
2.3.2.1 The principle of identity and decomposition

This principle allows the decision makers model a complex problem in a hierarchical structure of criteria, or factors (attributes), sub-criteria, and so on represented by random variables. They are organized in a hierarchy structure so that the factors that occur on higher levels of the hierarchy depend on lower level factors, in this way all influencing factors can be included and evaluated separately. An effective way to do this is first to work downward from the goal in the top level, to criteria in the second level, followed by sub-criteria in the third level, and so on, from the more general (and sometimes uncertain) to the more particular and definite. One can start at the bottom, identifying alternatives for that level and attributes under which they should be compared which fall in the next level up. In this way, one can link the top level (main goal) of the hierarchy to its bottom level (alternatives) in a sequence of appropriate intermediate levels (criteria).

A typical hierarchical structure for a decision making problem is with three main levels; the first level is the main goal, the second is the criteria and sub-criteria level and the third is the level of alternative options. A hierarchy is said to be complete when every element of a given level functions as a criterion and links with all elements of the level below. The hierarchy can be divided into sub-hierarchies with a common topmost element, the main goal.

As an example, Figure 2.1 shows an overall hierarchy with three main levels; the first level is the main goal of the problem. The second level is the level of the main criteria, with four criteria; namely A, B, C, D and then the third level, which is the level of alternative options, is with three alternative options; namely Option 1, Option 2, and Option 3.

Figure 2.1: Hierarchy Structure



2.3.2.2 The principle of comparative judgment

Once the hierarchy, interrelation between different criteria (decision factors), has been constructed the principle of comparative judgments is applied. Pair-wise comparison matrices, for each element at the same level, are constructed; these matrices contain the relative weights (priorities) of elements that are determined by comparing them in pairs, separately for each level in the hierarchy. The results for each level are recorded in a separate decision matrix. Hence, when comparing two criterions, the following must be explained:

1. Which criterion is more important or has greater influence on the criterion one level higher in the hierarchy;
2. The weight or intensity of that importance.

The relative priority (importance) of the elements in each level is determined by asking the decision makers include their subjective judgments to express their preferences over pairs of alternatives, based on various factors, included in the problem under consideration. Hence, pair-wise comparison between each of elements or objectives, which represent different criteria, sub-criteria, or alternatives at the same level, is applied. Each comparison is transferred in to a nine-scale value (Table 2.2), (Saaty, 1980); this implies that qualitative evaluations are converted into quantitative ones.

As an example: if a certain criterion (or sub-criterion) C with n elements in the next level below it: $A_1, A_2, ..., A_n$, is considered then a pair-wise comparison matrix can be explained as in Table 2.1.

Table 2.1: A pair-wise comparison (weighting) matrix

C	A_1	A_2	$\cdots$	A_n
A_1	a_{11}	a_{12}	$\cdots$	a_{1n}
A_2	a_{21}	a_{22}	$\cdots$	a_{2n}
$\cdots$	$\cdots$	$\cdots$	$\cdots.$	$\cdots$
A_n	a_{n1}	a_{n2}	$\cdots$	a_{nn}

where $a_{ij}, i, j = 1,..., n$ are the subjective judgments for a decision maker.

If w_1 is measured as a weight for A_1 and w_2 a weight for A_2, then to compare A_1 with A_2, the ratio w_1 / w_2 gives the relative value of measurements of A_1 to A_2 (the relative weight, ratio, for each pair). This has the advantage of focusing exclusively on two objects at the same time and on how they relate to each other. It also generates more information than is really necessary since each object is compared with every other.

Thus, the relation between the weights w_i and the judgments a_{ij} are simply given by: $a_{ij} = w_i / w_j$ for $i, j = 1,...,n$ and For example, if $i = j = 3$, the relation between the matrix representing the elements of the set $a_{ij}, i, j = 1,...3$ and the matrix of their corresponding weights can be expressed as

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} = \begin{pmatrix} w_1 / w_1 & w_1 / w_2 & w_1 / w_3 \\ w_2 / w_1 & w_2 / w_2 & w_2 / w_3 \\ w_3 / w_1 & w_3 / w_2 & w_3 / w_3 \end{pmatrix} \tag{2.2}$$

Hence, for each judgmental matrix A, $a_{ij} = w_i / w_j$ for $i, j = 1,...,n$, represents the pair-wise comparison of element A_i over A_j with respect to criterion C, $w_i (i = 1,...n)$ are the derived scale values represent their corresponding weights or intensities with:

$a_{11} = a_{22} = ... = a_{nn} = 1$ and

$a_{ij} = 1/a_{ji}$ for $i, j = 1,2,...,n$.

Thus, AHP always deal with positive reciprocal matrices and need only conclude $n(n-1)/2$ judgments where n is the total number of elements being compared. With traditional AHP, a nine-point scale (1-9) is used for entering judgments. (Saaty, 1977; 1980; 1990; 2001; Lin et al., 2008). Table 2.2 explains details for the 9-scale measurements.

Table 2.2: The pair-wise comparison scales

Scale of Importance	Definition	Explanation
1	Equal importance	Two factors equally preferred
3	Weak importance	One factor compare to the other affect the goal slightly (weakly preferred)
5	Strong important	One factor strongly affects the goal compare to the other factor (strongly preferred)
7	Very strongly importance	One factor is favoured strongly over the other (very strongly preferred)
9	Extremely important	One factor is extremely in favour of the other (absolutely preferred)
2,4,6,8	Intermediate values	when compromise is needed

These values, 1-9 and their reciprocals, correspond to strength of preference of one element over another, are used for each comparison to set up a judgmental matrix to carry out pair-wise comparisons of the relative importance of the elements in the second level with respect to the overall focus (main goal) of the first level. For example, a scale of "1" indicates equal important of two elements contributing to the upper level element; "9" indicates relative importance of one element over another and a value between "1" and "9" provides importance measurements of one element over the other. Additional comparison matrices are used to compare the elements of the third level with respect to the appropriate parents in the second, and so on down the hierarchy. At each level the relative importance of each factor is assessed by comparing them in pairs, and then the rankings (priorities), obtained through the pair-wise comparisons between the different factors, are converted to normalize ranking using the Eigen-value method. The pair-wise comparison, reflects the estimates made by the decision makers of the relative importance of each alternative in terms of a given decision criteria, is used to generate a derived ratio scale. The comparison results are then composed in to a positive reciprocal matrix $A = (a_{ij})$, where $a_{ij} = a_{ik}.a_{kj}$ (Table 2.1). Where each $a_{ij} = w_i / w_j$ for $i, j = 1,...,n$.

This builds a reciprocal paired comparison matrix and its corresponding principal eigenvector. The next step deals with composition of the derived ratio scale.

2.3.2.3 The principle of composition or synthesis

When the pair-wise comparison matrices has been established, relative weights are calculated for each set of elements at every level of hierarchy and their respective priority vectors are determined, then the principle of hierarchic composition or synthesis is applied, where the overall score of each alternative, representing the preference of one alternative over another is calculated. An Eigen-value technique, from principle of linear algebra, is employed to calculate the weights of overall relative priorities for each alternative. Priorities are synthesized from the second level down by multiplying local priorities by the priority of their corresponding criterion in the level above, and adding for each element in a level according to the criteria it affects. This calculation, starts at the lowest level and ends at the primary goal level, gives the global priority of that element, which is then used to weight the local priorities of the element as the criterion, and so on to the bottom level. According to linear algebra, the largest Eigen-value of the pair-wise matrix, $\lambda_{\max}$ can be calculated from the Eigen-value technique defined by the following equation:

$$\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{pmatrix} = \lambda_{\max} \cdot \begin{pmatrix} w_1 \\ w_2 \\ \cdots \\ w_n \end{pmatrix} \tag{2.3}$$

where a_{ij} are defined by equation (2.2) and $w = (w_1, w_2,..., w_n)^T$ is the priority vector for the pair wise matrix.

Although, every matrix has a set of Eigen-values and for every Eigen-value there is a corresponding Eigen-vector, Saaty (1980; 2003) showed that a pair-wise comparison judgemental matrix can have just one Eigen-value and that the correspondent Eigen-vector, normalized to 1, can be a very good measure of the relative importance of the alternatives, and that the principle Eigen-vector is a necessary representation of the priorities derived from a positive reciprocal pair-wise comparison judgment matrix. Therefore, the Eigen-vector is called the priority vector of the elements compared, which represents their relative weights with regard to the objectives located one level higher in the hierarchy. Furthermore, the lambda max $\lambda_{\max}$ technique was utilized to determine the final weights of the criteria in the pair-wise comparison method. With Lambda Max technique, a vector of weights is defined as the normalized Eigen-vector corresponding to the largest Eigen-value $\lambda_{\max}$. Finally, by a linear additive aggregation procedure, the total priority of each criterion relative to the overall objective is derived based on all the generated weights.

2.3.3 Consistency Ratio

The AHP method described above is an effective and structured method for evaluating the relative ratio scale or weights for various alternatives. However, in addition to the limitation of the nine-scale points, AHP allows subjective judgments play an important role in the selection process, then inconsistency of decision maker's judgments during the pair-wise comparison process is expected and the Eigen-vectors representing the entire weight vectors might be invalid, then perfect consistency is practically difficult to achieve. The inconsistency within AHP is described by Saaty (1980) as perturbation in the coefficients of the matrix A which can be represented by the difference between number of criteria n and the largest Eigen-value $\lambda_{\max}$. Then, it is shown that the reciprocal matrix A with positive entries is consistent if and only if $n = \lambda_{\max}$ (Lin at el, 2008). The ability of AHP to test for consistency is one of the method's greatest strengths. It is based on the idea of transitivity, which means that if A is twice as important as B and B is three times as important as C then A should be six times as important as C.

Therefore, for controlling the consistency of pair-wise comparison, Saaty developed a tool called Consistency Ratio (CR); this measures the degree of consistency among the pair-wise judgments before proceeding with the final analysis and enables decision makers to control the extent of inconsistency, to a maximum desirable level, for the entire hierarchy. To measure CR, it is necessary to estimate Consistency Index (CI) which is defined as

$$CI = \frac{\lambda_{\max} - n}{n-1} \tag{2.3}$$

where n is the number of criteria for each level.

Hence, the consistency of the comparison matrix is determined by the CR, which is obtained by comparing the determined value for CI, from (2.3), and the Random Inconsistency Index RI (Saaty, 1980), i.e.,

$$CR = \frac{CI}{RI} \tag{2.4}$$

Few values for those RI are listed in Table 2.3 (Saaty, 1980; 2001; Saaty and Vargas, 2006).

Table 2.3: Random Inconsistency Index for different numbers of criteria

Number of criteria	2	3	4	5	6	7	8	9	10
RI	0	0.52	0.89	1.11	1.25	1.35	1.40	1.45	1.51

In this table (Table 2.3), Saaty (1995) explained that a matrix said to be consistent with acceptable judgmental consistency if, for example, the number of criteria is 4 then the RI value is given to be 0.89.

However, perfect consistency cannot be demanded since human beings are often biased and inconsistent in their subjective judgments; Saaty (1980) shows that if CR is less than 0.10, or if the CR value is sufficiently small then the degree of inconsistency is satisfactory then the judgments are probably consistent enough to be useful; otherwise, inconsistent judgments indicating that the decision maker needs to revise the original values in the pair wise comparison.

In this research study, these results, by Saaty, have been applied to all pair-wise comparisons for the judgmental matrices in AHP applications.

Later in the next chapters, the AHP, combined with the other preference ranking techniques, is used as a powerful decision making tool, to develop a new methodology to propose an integrated AHP-based model for ranking alternative options for financial investment scenarios along with applications to the real world problems.

2.4 Summary

In this chapter, some particular techniques, on which this thesis is based on, are considered with their main concepts, theories, and background information; these are necessary for the development of the new approaches that are introduced in this thesis. First, the basic concepts for EUT and its axioms are studied; these will be used to introduce, for decision making problems under uncertainty, the simplest investment model for financial economics; namely the MVM. Hence, MVM, which will be based on for conducting a new methodology for ranking preferences to solve decision problems, is reviewed and its main concepts are explained, in addition its consistency with EUT is introduced. Later in this chapter, an AHP model, which will be considered in establishing an integrated AHP-based model, and its main concepts and hierarchy construction are introduced; it is explained how it can be used as a flexible tool that offers decision makers the possibility of assigning different values to non-monetary factors and helps to building a cost benefit model that uses qualitative in addition to quantitative variables.

CHAPTER THREE: A NEW PREFERNCE RANKING MODEL AND ANALYSIS

In Chapter 2, theories and background information that concern each of Expected Utility Theory (EUT) and Mean Variance Analysis (MVA), which are related to this study, were introduced and analysing preferences under risk in the context of EUT was explained. Following this, Mean Variance Model (MVM) was explained, which can rank alternative options, representing financial scenarios, according to the value of a function defined over the first two moments the mean and the variance. In this chapter, a novel approach for ranking pairs of alternative options, with only non-negative outcomes, is proposed; it is based on reformulating MVM to be based on cumulative function using simulation. In this new development, a new generalized approximation framework for analysing preferences is established; it is not only capable of tackling the shortcomings of MVM, but also can solve large-scale decision problems especially those encountered with financial scenarios

3.1 Introduction

A decision-making problem under uncertainty can be regarded as a problem involving a choice between options represented by random variables (lotteries) according to a preference ordering. The two theories that have received broad application in analyzing preferences in the field of economics are the application of EUT and MVA, which were explained earlier in Chapter 2.

Despite that the MVM, introduced by MVA, plays an important role in analysing preferences for various alternatives, this model is appropriate only when random variables representing the alternative options are normally distributed and the utility function to be maximized is quadratic; both properties limit the applicability of the models in real life applications. In this chapter, a new approach, a reformulation of MVM, that overcomes the limitations of the existing model, based on cumulative distribution functions and using simulation, is proposed. The new model, formulated under this approach, can accommodate a variety of randomly distributed variables, representing financial investment options for risk-averse investors, with different utility functions. The formulation of this model is described in the following sections.

3.2 Model based on MVA and Cumulative Distribution Function using Simulation

The aim of the study, in this chapter, is to introduce a new ranking strategy for analysing preferences for pairs of alternative options that concern financial scenarios, financial investment for risk-averse investors is an example. Based on the approximation results of Levy and Markowitz (1979) and Hlawitschka (1994), relying on cumulative functions, MVM is reformulated to propose a new approximation model that can overcomes the deficiency of the existing model, using simulation.

The new approach, the reformulated MVM, is based on cumulative distribution function; instead of requiring a random variable X representing a lottery with probability distribution functions $f(x)$, another random variable, which is called the cumulative random variable, denoted by F_X, which is associated with the cumulative distribution functions $F_X(x)$, is required. Based on the main property for cumulative distribution functions $F(x)$, $0 \le F(x) \le 1$, the inverse cumulative function F^{-1} can be determined. This implies that, different random values representing the real-world random variables from the inverse function can be generated. This can be explained by the following example.

Example 3.1: Suppose that a random variable X has an exponential probability distribution function $f(x)$, $f(x) = e^{-x}$ for all $x > 0$.

Then, the cumulative function $F(x)$ for X is determined as $F(x) = 1 - e^{-x}$ for all $x > 0$

Hence, the cumulative function $F(x)$, in Example 3.1, in terms of the cumulative random variable F_X, can be written as $F_X(x) = 1 - e^{-F_X}$ for $x > 0$, F^{-1} can be determined by solving the equation $F(x) = r$, $r \in R(0,1)$. This implies that $x = -\ln(1-r)$, so $F^{-1}(r) = -\ln(1-r)$. If R has a uniform distribution function $R(0,1)$, then the cumulative random variable F_X, which is defined by $F_X = F^{-1}(R) = -\ln(1-R)$, has an exponential distribution. In practice, as both, $1-R$ and R have a $R(0,1)$ distribution, $1-R$ can be replaced by R. This implies that a variety of random values, from the inverse function, can be generated.

Levy and Markowitz (1979) and Hlawitschka (1994) show that MVM can be regarded as a second order Taylor series approximation of standard utility functions such as the power or the exponential utility function. Hence, based on the same approximation results obtained by equation (2.1), by assuming power, linear power or exponential utility function defined over the real line, to propose a new approach for MVM based on cumulative functions; results from both Levy and Markowitz and Hlawitschka can be followed. However, the preference ordering over a set of alternative options can be represented by a set of lotteries, which can be considered as random variables taking values on the real line and random variables can be expressed or described via cumulative distribution functions. Hence, comparison between different random variables can be approached by examining their associated cumulative functions which are increasing functions and possess inverses.

Therefore, if a pair of lotteries (X, Y) on the real line is considered, then the cumulative random variable F_X is defined as a random variable associated with $F_X(x)$ and the cumulative random variable F_Y is defined as a random variable associated with $F_Y(x)$. Consequently, for the ranking process, the cumulative random variable F_X for the distribution functions $F_X(x)$ and the cumulative random variable F_Y for $F_Y(x)$, rather than the lotteries can be used and preferences over lotteries can be thought of as preferences over such cumulative random variables. The problem is then to establish a preference ordering over a set of such distribution functions or over the cumulative random variables. If the ordering is consistent in the sense of Von-Neumann and Morgenstern (VNM) utility theory, then it can be represented by a utility function $U(x)$ so that $\int_{-\infty}^{\infty} U(x)dF_X(x) \geq \int_{-\infty}^{\infty} U(x)dF_Y(x)$ if and only if F_X is ranked above F_Y in the preference ordering. From equation (2.1), $E[U(F_X)]$ can be expressed in terms of the two parameters, the mean and the variance for F_X.

Similarly, $E[U(F_Y)]$ can be expressed in terms of the two parameters, the mean and the variance for F_Y. Then an investor's preference over alternatives can be determined, and the alternative with higher expected utility is preferred more.

Moreover, from the dominance property, it can be showed that for any pair of lotteries, if a cumulative variable F_X representing the first lottery dominates second cumulative variable F_Y representing the second lottery then the total value for cumulative function $F_X(x)$ associated with the first lottery is less than that for the cumulative function $F_Y(x)$ for the second lottery F_Y. i.e. If F_X dominates F_Y then $F_X(x) < F_Y(x)$. This can also be verified graphically.

The preference relationship between lotteries in P and their cumulative variables is assumed as follows:

Definition 3.1: For any pair of lotteries $(X,Y) \in P$, (P is the set of all lotteries), X is preferred to Y $(X \underset{P}{\succ} Y)$, is equivalent with $F_X \underset{P}{\succ} F_Y$. F_X is the cumulative random variable associated with cumulative function $F_X(x)$ and F_Y is the cumulative random variable associated with cumulative function $F_Y(x)$. $\succ_P$ denotes a binary preference relation on the set P, analogous to the preference relation defined by expected utility theory, and $F_X(x) = pr\{X \le x\}$ for all $x \in$ **Re**, the real line.

Definition 3.1 explains that, the preferences over lotteries associated with the probability distribution functions are equivalent with the preferences over the cumulative random variables associated with cumulative distribution functions.

Hence, from the results of Taylor series approximation proposed by Levy and Markowitz (1979) and Hlawitschka (1994), which is given by equation (2.1), from the previous chapter, the expected utility for each cumulative random variable, F_X for example, is

$$E[U(F_X)] = U(\mu_{F_X}) + \frac{U''(\mu_{F_X})}{2} * \sigma^2{}_{F_X} \tag{3.1}$$

Where U is the VNM utility function (defined over real line **Re**) that represents preference ordering over any pair (F_X, F_Y) of cumulative random variables. μ_{F_X} represents the mean and $\sigma^2{}_{F_X}$ represents the variance for the cumulative variable F_X, $U'' < 0$ is the second derivative for the utility function around μ_{F_X}.

Theorem 3.1: Let the VNM utility function U (defined over real line **Re**) represents preferences over cumulative random variables (lotteries) F_X and F_Y , $F_X \succ_P F_Y$ if and only if $E[U(F_X)] \ge E[U(F_Y)]$.

Proof: Suppose that F_X, F_Y are defined over a closed interval $[a,b] \in$ **Re**

First, to show that, if $F_X \succ_P F_Y$ then $E[U(F_X)] \ge E[U(F_Y)]$, From dominance condition, first order stochastic, If F_X dominates F_Y then $F_X(x) \le F_Y(x)$ for all $x \in [a,b]$.

Thus, $[F_X(x) - F_Y(x)] \le 0$, for all $x \in [a,b]$.

Now consider the following:

$$\int_a^b u(x)[d(F_X(x) - F_Y(x))] \tag{3.2}$$

Integrating equation (3.2) by parts, we obtain

$$\int_a^b u(x)[d(F_X(x) - F_Y(x))] = u(x)[F_X(x) - F_Y(x)]_a^b - \int_a^b u'(x)[F_X(x) - F_Y(x)]dx$$

As $[F_X(x) - F_Y(x)]_a^b = 0$ and $u'(x) > 0$, then

$$\int_a^b u(x)[d(F_X(x) - F_Y(x))] = -\int_a^b u'(x)[F_X(x) - F_Y(x)]dx,$$

Hence, $[F_X(x) - F_Y(x)] \le 0$, which implies that $\int_a^b u(x)[d(F_X(x) - F_Y(x))] \ge 0$.

If $\int_a^b u(x)dF_X(x) \ge \int_a^b u(x)dF_Y(x)$ then $E[U(F_X)] \ge E[U(F_Y)]$

Conversely, if $E[U(F_X)] \ge E[U(F_Y)]$ then it is possible to show that F_X is preferred over lottery F_Y.

If $E[U(F_X)] \ge E[U(F_Y)]$ then $\int_a^b u(x)dF_X(x) \ge \int_a^b u(x)dF_Y(x)$, this implies that

$\int_a^b u(x)[d(F_X - F_Y)] \ge 0$. Solving the last integral by parts implies that

$-\int_a^b u'(x)[F_X(x) - F_Y(x)]dx \ge 0,$ $u'(x) > 0$, and then $F_X(x) \le F_Y(x)$.

Thus, from dominance condition, this is satisfied if $F_X \underset{P}{\succ} F_Y$, and then the desired result is obtained. ♦

Remark 3.1: Theorem 3.1 explains that, when the expected utility for each cumulative random variable is determined then the option with higher expected utility is preferred more.

3.2.1 The proposed model using simulation

In this section, the new proposed model, a combination of the two consistent models, MVM and EUM, for ranking pairs of alternative options is explained. Instead of considering random variables with their probability functions, cumulative random variables with their cumulative distribution functions are considered. Using simulation, the inverse transformation method generates different random values from the inverse cumulative function of each cumulative random variable to stand for the random variable that represents the uncertain alternative.

Basically, the inverse transformation method depends on the properties of $F(x)$, the cumulative distribution function of a random variable X and random numbers which are uniformly distributed between 0 and 1. Since $F(x)$ is a non-decreasing function and F^{-1} exists, then every value of x has a unique value $F(x)$ associates with it. If a random value of $F(x)$ is considered, which is between 0 and 1, then an associated value of x, either explicitly or by using a computational algorithm can be identified; x values are random values from the probability distribution function $f(x)$. Thus, the cumulative function $F_X(x)$, $0 \leq F_X(x) \leq 1$, possesses the same property as the uniform random numbers. Therefore, to generate a random value from a cumulative random variable, a uniform random number $r \in R(0,1)$ is only needed to set it equal to the cumulative distribution function, then solve for x.

Now, based on the inverse transformation method and the properties of cumulative function $F_X(x)$ with the uniform random numbers, it is possible to present the following:

Theorem 3.2: If $F_X(x)$ is the cumulative distribution function associated with the random variable F_X, then for each random value x from F_X, there exists a random number $r \in R(0,1)$, so that $x = F^{-1}(r)$ if and only if $F(x) = r$.

Proof: Consider $F_X(x)$, assume that for each x there exists a real number $r \in R(0,1)$ such that $x = F^{-1}(r)$. This implies that, $r \leq F(x)$ is equivalent with $F^{-1}(r) \leq x$.

Instead of r, if a random variable R defined over $R(0,1)$ is considered then it is implied that their corresponding events are the same:

$$\{R \leq F(x) = F^{-1}(R) \leq x\} \tag{3.3}$$

Assume that a uniform random variable $R(0,1)$ is considered, then $P(R \leq b) = b$, where b is any constant, $0 \leq b \leq 1$. If $b = F(x)$ then $P(R \leq F(x)) = F(x)$.

Hence, from equation (3.3), it implies that $P(F^{-1}(R) \leq x) = F(x)$. In other words, the random variable $F^{-1}(R)$ has distribution function $F(x)$.

Therefore, for each $r \in R(0,1)$ there exists an x such that $F(x) = r$.

Conversely, if $F(x) = r$ then the inverse function for $F(x)$ exists. Thus, $F^{-1}(F(x)) = F^{-1}(r)$ implies that $x = F^{-1}(r)$. ♦

Remark 3.2: Theorem 3.3 is derived from inverse transformation method, based on the analogous properties of $F(x)$ and random numbers; with no restrictions on the distribution function $F(x)$.

Therefore, if the cumulative random variable F_X, which is associated with the cumulative function $F_X(X)$, is considered then different random values, from the inverse function F^{-1} of this cumulative function, can be generated. In order to determine the values for the mean and the variance for F_X, the mean and the variance for the generated values are determined to represent these values. Hence, from equation (3.1) the expected utility for F_X can is computed.

Similarly, randomly selected values are generated for cumulative random variable F_Y, which represents the second lottery, and then the mean and the variance are determined. Hence, from equation (3.1), the expected utility for F_Y is determined. Finally, the most preferred lottery will be the one with higher expected utility, higher mean and less variance for most of the generated values.

Hence, the new proposed preference ranking model, which is based on MVM and cumulative distribution function using simulation, is described by the following:

3.3 Algorithm and Implementations

This section introduces an algorithm that describes the proposed model for ranking preferences for pairs of alternative options with non-negative outcomes; it is based on MVM and cumulative distribution function, using a simulation method called inverse transformation method. The use of simulation is to generate different random values to be representative for each of the generated random variables (lotteries) F_X, F_Y from the inverse cumulative distribution functions F^{-1}. Hence, for each of such randomly generated variables, the algorithm determines the mean, the variance and the expected utility, and then the preference ordering for each pair of generated variables is determined as detailed below.

3.3.1 Algorithm 3.1

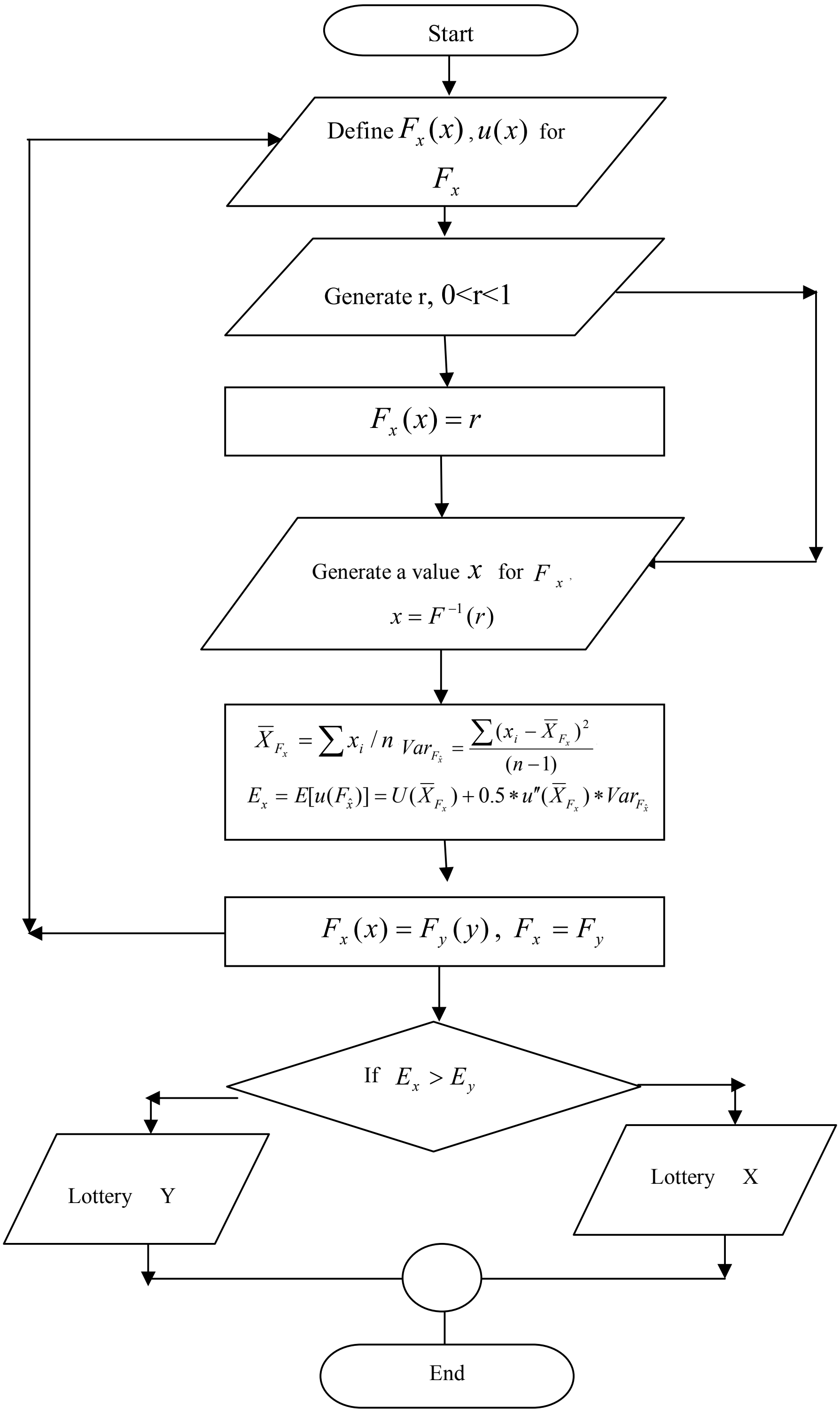
The main procedure for this algorithm, Algorithm 3.1, is defined by the following main steps:

1. Define the cumulative distribution function $F_X(x)$ for the random variable F_X which posses a utility function $u(x)$;
2. Generate random numbers r, $0 < r < 1$ from uniform distribution function;
3. Equate $F_X(x)$ to r;
4. Generate the first value x for F_X from the inverse function F^{-1}, $x = F^{-1}(r)$;
5. Repeat 2-4 to generate different values for F_X;
6. Determine the mean and the variance for the generated values of the random variable F_X, then from equation (3.1) determine the utility for the computed mean and the expected utility for F_X using a specific utility function ;
7. Repeat 1-6, by considering the random variable F_Y with cumulative function $F_Y(x)$;
8. The preferred lottery is the one with the higher expected value, higher mean with least variance for most of the generated values.

Remark 3.3: Algorithm 3.1 is considered as the key element to describe the main steps for the new proposed ranking model. With this algorithm, pairs of cumulative random variables are generated to represent different uncertain alternative pairs, and then more preferred option for each pair is determined.

Flow Chart 3.1 illustrates the main steps of Algorithm 3.1.

Flow Chart 3.1: Algorithm 3.1 flow chart



For the implementation of the algorithm, each simulation run consists of a specified number of replications denoted by n, for each replication within those simulation runs, by using a simulation routine, uniform random numbers are generated. Using these numbers, different values for these random variables from their inverse cumulative distribution functions are generated to represent the simulated random variables; each stands for an alternative option. Hence, the mean, the variance and the expected utility for each of the generated variable is determined, according to some specific utility function and for some specific values of parameters, which are given by the user. Later, based on the determined expected utility, ranking of alternatives is determined. The most preferred option will be the one with higher expected utility, higher mean and less variance.

3.3.2 Implementation and simulation results

For the implementation of Algorithm 3.1, the simulation procedure was implemented in MATLAB, using inverse routines of different cumulative functions, pairs of random variables were generated, especially normally distributed pairs defined for different utility functions such as power, exponential and linear exponential functions. Ten thousand replications were used for most simulation runs; with this number the running time was found to be efficient. Several cases were run using $n = 100{,}000$ replications and the results were almost the same; while larger n led to overtime running.

Tables, Table 3.1-3.5 summarize results for some specific simulation runs that generate pairs of random variables from their inverse cumulative functions, especially normally distributed, to represent a pair of uncertain alternatives, using different utility functions such as power, exponential or logarithmic function for different parameters, to be used for ranking process; each is run for a number of replications.

Figures, Figure 3.1-3.5, explain the graphical representation for the cumulative functions for the generated values that represent the real cumulative variables.

The obtained results from the simulation runs can be summarized as follows:

1) Table 3.1 explains that a pair of normally distributed lotteries X, Y is considered with a utility function, which is defined to be a power function, i.e.

$$U(x) = \frac{x^{(1-\theta)}}{(1-\theta)}, \quad 0 < x \le 1 \text{ and } 0 < \theta < 1 \text{ with } U''(x) = -\theta * x^{(-\theta-1)} < 0$$

The algorithm is run for n replications; n takes different values, at most $n = 100000$, and the parameter θ for the power function equals 0.5.

In this table, Table 3.1, X and Y are assumed to be normally distributed with different means ($\mu_X = 2, \mu_Y = 10$) and equal variances ($\sigma_X = 1, \sigma_Y = 1$), i.e. $X \sim N(2,1)$ and $Y \sim N(10,1)$. The utility function is defined to be a power function with the parameter $\theta = 0.5$.

For this table, for the first lottery X, the approximated values for mean, variance, expected utility, and the total cumulative, from the simulation results, are determined as follow:

The mean $\mu_{XSimulated} = 2.000$, The variance $Var_{XSimulated} = 1.006$

The expected utility $E[U(X_{Simulated})] = 2.739$

The total cumulative value for $X_{Simulated} = 0.999$

For the second lottery Y, the obtained values are:

The mean $\mu_{YSimulated} = 9.992$, The variance $Var_{YSimulated} = 0.993$

The expected utility $E[U(Y_{Simulated})] = 6.315$

The total cumulative value for $Y_{Simulated} = 0.991$

The obtained results show that the second lottery, Y with higher expected utility, is more preferred to the first lottery X.

Moreover, Figure 3.1 illustrates that the total cumulative value for simulated Y is less than that for the simulated X, hence it is more preferred to X, this explains the dominance condition, the first stochastic dominance, If Y dominates X then $F_Y(x) < F_X(x)$.

While running the algorithm for different mean values but the same values for the variance, for various numbers of replication, the same ranking preferences are obtained. In addition, the results are almost the same when various utility functions such as exponential or linear exponential for normally distributed random variables are used.

2) Table 3.2 introduces the simulation results for the same application where the two random variables are normally distributed, with the same means but different variances, and a different parameter value for the utility function $\theta = 0.9$. The simulation results show that the alternatives are similarly ranked.

Moreover, Figure 3.2 brings up a similar preference ordering as for results in Table 3.2.

3) Table 3.3 shows results for simulation runs for the same algorithm but for other values of parameters, random variables which are normally distributed with the same means but different variances. The utility function is defined to be a power function with $\theta = 0.5$. The simulated results develop a similar ranking preference.

Remark 3.4

Tables 3.2-3.3 display significant conclusion for the applicability of the new proposed model; it is the possibility of using it for ranking preferences if only variances are considered. However, studies have shown that ranking alternatives are not always possible if only lottery variances (risks) are considered. For example, Sarin and Weber (1993) ensure that: the variance, which is the measure of risk, cannot be used as the basis for ranking alternatives, regardless of their distributions and show examples that can be constructed where an option with higher mean and lower variance may not be preferred by risk-averse decision maker. Despite that the simulation results that are obtained from the implementation of Algorithm 3.1 would, graphically (Figure 3.3), support the same conclusion as for those studies, which shows that for any pair of lotteries (F_X, F_Y) with equal means and different variances, it is not always possible to obtain the dominance condition with $F_X(x) < F_Y(x)$, for all x; results in Table 3.3 reveal a significant conclusion, which is the identification of the most preferred lottery as the one with less variance while the lottery means are equal.

4) Table 3.4 concludes similar results, for the simulation runs of the algorithm, as in Tables 3.1-3.3, but with different utility function, which is chosen to be logarithmic function. Figure 3.4 shows, graphically, the same results as for figures Figure 3.1- 3.2.

5) With Table 3.5, the algorithm is applied to a pair of lognormal random variables and a logarithmic utility function and with almost the same results as obtained by Levy (1992), the variance is a legitimate measure of risk if the utility function is increasing and concave and random variables have lognormal distribution. It is shown that when the means are equal, the algorithm can give satisfactory results if the random variables are lognormal. Graphically from Figure 3.5, the same results are obtained as those from Figure 3.4.

In addition, similar results, as shown in Tables 3.1-3.5, are obtained when a variety of other simulation cases are studied that involve other parametric values for the parameters of random variables and the utility functions with different probability functions.

Remark 3.5

Empirical evidences from simulation results ensure that the new proposed algorithm that describes the new approximation model provides a preference basis; it can easily deal with normal random variables with equal means and different variances or difference means and equal variances subjected to many desirable utility functions such as quadratic, exponential and linear plus exponential utility function, with different parameters. This can be compatible with some other models and support their findings, in addition it resolve the shortcomings of others, in dealing with different random variables and utility function.

The following tables, Table 3.1-3.5, show the obtained simulation results for the simulation runs with specified number of replications and for different values of parameters.

Table 3.1: Normal random variables X, Y with utility function defined as a power function

Parameter Values	Simulation Results
$n = 10{,}000$ $\theta = 0.5$	$\mu_{XSimulated} = 2.000$ $Var_{XSimulated} = 1.006$ $E[U(X_{Simulated}) = 2.739$ $Cumulative(X_{Simulated}) = 0.999$
$\mu_X = 2.0$ $Var_X = 1.0$	
$\mu_Y = 10.0$ $Var_Y = 1.0$	$\mu_{YSimulated} = 9.992$ $Var_{YSimulated} = 0.993$ $E[U(Y_{Simulated}) = 6.315$ $Cumulative(Y_{Simulated}) = 0.991$

Table 3.2: Normal random variables X, Y with utility function defined as a power function

Parameter Values	Simulation Results
$n = 10{,}000$ $\theta = 0.9$	$\mu_{XSimulated} = 2.015$ $Var_{XSimulated} = 1.016$ $E[U(X_{Simulated}) = 10.604$ $Cumulative(X_{Simulated}) = 0.999$
$\mu_X = 2.0$ $Var_X = 1.0$	
$\mu_Y = 2.0$ $Var_Y = 5.0$	$\mu_{YSimulated} = 9.992$ $Var_{YSimulated} = 4.987$ $E[U(Y_{Simulated}) = 8.805$ $Cumulative(Y_{Simulated}) = 0.987$

Table 3.3: Normal random variables X, Y with utility function defined as a power function

Parameter Values	Simulation Results
$n = 10{,}000$ $\theta = 0.5$	$\mu_{XSimulated} = 2.000$ $Var_{XSimulated} = 0.998$ $E[U(X_{Simulated}) = 2.739$ $Cumulative(X_{Simulated}) = 0.980$
$\mu_X = 2.0$ $Var_X = 1.0$	
$\mu_Y = 2.0$ $Var_Y = 4.0$	$\mu_{YSimulated} = 1.999$ $Var_{YSimulated} = 3.993$ $E[U(Y_{Simulated}) = 2.495$ $Cumulative(Y_{Simulated}) = 0.999$

Table 3.4: Normal random variables X, Y with utility function defined as a Logarithmic function

Parameter Values	Simulation Results
$n = 10{,}000$	$\mu_{XSimulated} = 2.003$ $Var_{XSimulated} = 1.001$ $E[U(X_{Simulated}) = 2.580$ $Cumulative(X_{Simulated}) = 0.999$
$\mu_X = 2.0$ $Var_X = 1.0$	
$\mu_Y = 4.0$ $Var_Y = 1.0$	$\mu_{YSimulated} = 4.016$ $Var_{YSimulated} = 1.010$ $E[U(Y_{Simulated}) = 3.945$ $Cumulative(Y_{Simulated}) = 0.989$

Table 3.5: Lognormal random variables X, Y with utility function defined as a logarithmic function

Parameter Values	Simulation Results
$n = 10{,}000$	$\mu_{XSimulated} = 1.872$ $Var_{XSimulated} = 0.965$ $E[U(X_{Simulated}) = 0.352$ $Cumulative(X_{Simulated}) = 0.808$
$\mu_Y = 0.5$ $Var_Y = 0.5$	
$\mu_Y = 0.5$ $Var_Y = 1.0$	$\mu_{YSimulated} = 2.756$ $Var_{YSimulated} = 1.800$ $E[U(Y_{Simulated}) = -0.673$ $Cumulative(Y_{Simulated}) = 0.969$

The following figures, Figure 3.1-3.5, represent the graphical representation for the cumulative functions for the simulated values given by the tables, Tables 3.1-3.5, respectively.

Figure 3.1: **Cumulative functions for simulated normal random variables with utility function defined as a power function, Table 3.1**

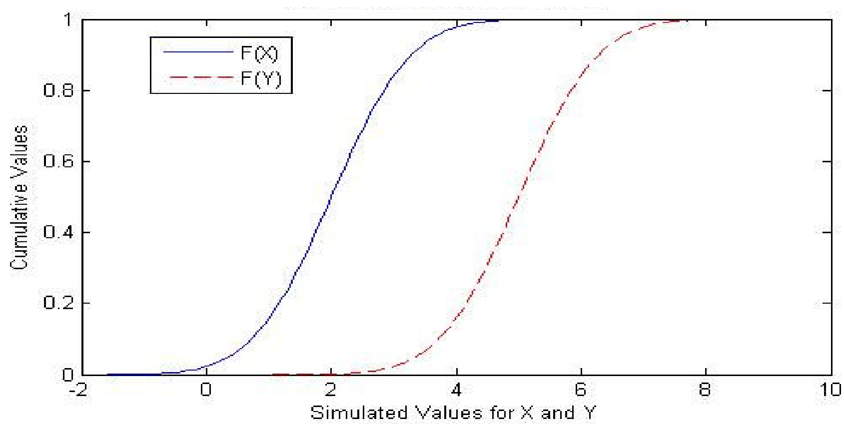


Figure 3.2: **Cumulative functions for simulated normal random variables with utility function defined as a power function, Table 3.2**

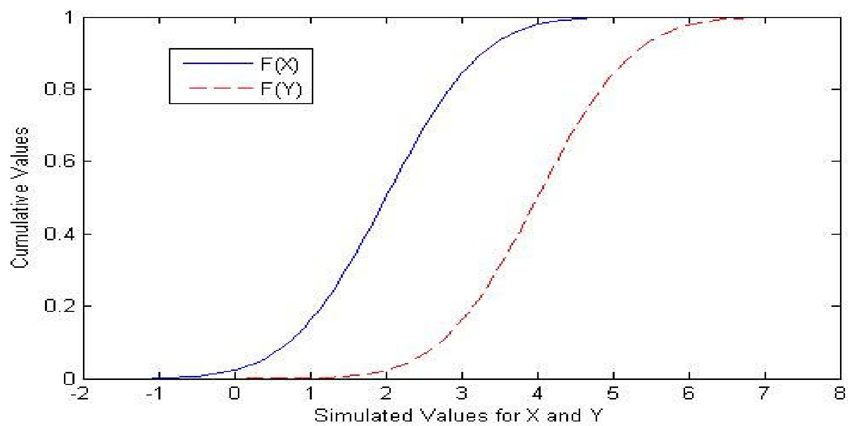


Figure 3.3: **Cumulative functions for simulated normal random variables with utility function defined as a power function, Table 3.3**

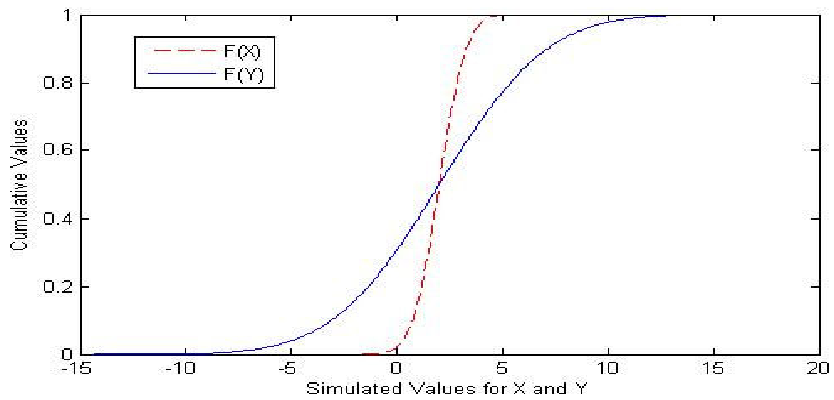


Figure 3.4: **Cumulative functions for simulated normal random variables with utility function defined as a Logarithmic function, Table 3.4**

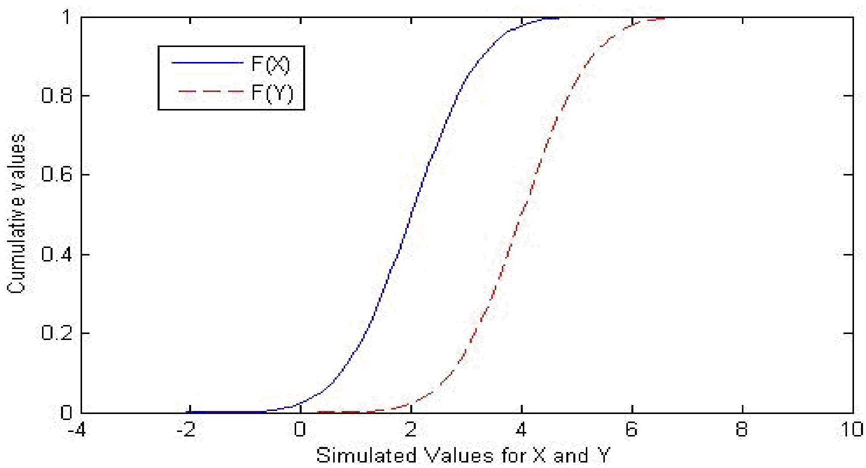
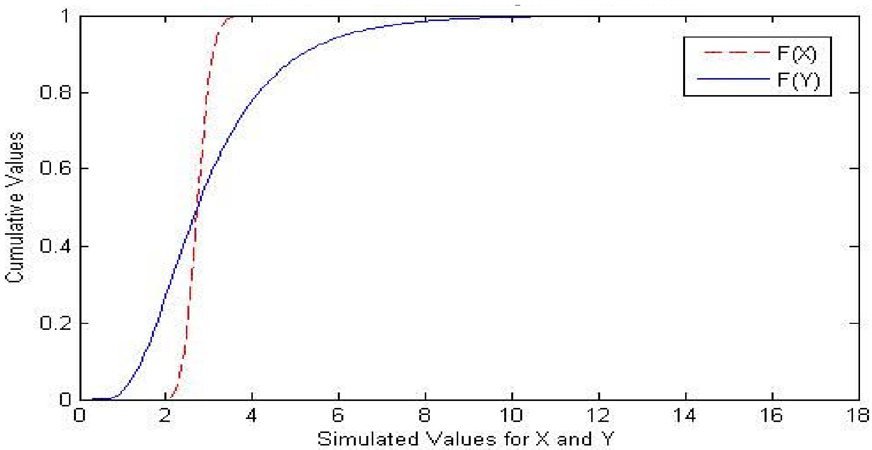


Figure 3.5: **Cumulative functions for simulated lognormal random variables with utility function defined as a logarithmic function, Table 3.5**



3.4 Summary

In this chapter, a novel approach, a reformulated MVM based on cumulative distribution functions, for ranking preferences was proposed. A new algorithm, using simulation, was introduced; it represented uncertain alternative by generating different random values from the inverses of the cumulative functions. The mean, the variance and the expected utility for each of the generated random variables were computed. Finally, the preference ordering for each pair of generated random variables, representing pairs of alternatives, was determined; the most preferred alternative was the one with highest expected utility, higher mean and less variance (almost always).

The new preference model, which was proposed under this approach, was used for ranking pairs of lotteries, with non negative outcomes, representing financial scenarios, for example financial investment options for risk-averse investors. In this new development, a new generalized approximation framework for analysing preferences was established. The model proposed under this approach was found to be very flexible in analysing preferences; it was not necessary to impose any restrictions on the distribution functions for the random variables. It could be applied to differently distributed random variables with no difficulty in considering the normal random variable, with a variety of utility functions according to the application fields of this model, such as exponential, linear exponential, logarithmic or any other distribution functions.

Moreover, the implementation of the proposed algorithm has shown desirable results; it has demonstrated that, in decision making under uncertainty, ranking preferences for pairs of lotteries, regardless of their distributions, can be obtained even if only lottery variances (risks) are considered. Based on the simulated results (Table 3.3) the most preferred option can be identified as the one with less variance while the means are equal. The results will be used in the following chapter for constructing a flexible risk-preference model that will help decision maker rank normalized lotteries with equal means. Furthermore, the model framework, specifically (Table 3.5), supports the use of lognormal random variables, lotteries, with an increasing and concave utility function; if the means are equal then the variance, a measure for risk, can be used as an appropriate measure for analysing preferences. Thus, the new preference ranking methodology introduced in this chapter would be applicable for solving a large scale of decision making problems under uncertainty in the field of economics; financial investment scenario is an example.

CHAPTER FOUR: RISK-AVERSE PREFERENCE MODEL

In this chapter, assuming that risk is an important component in the decision problems, based on the main results that were obtained from the previous chapter, *Chapter 3*, which ensured the possibility of ranking preferences for pairs of lotteries based on their variances while the lottery means are equal, another reformulation for Mean Variance Model (MVM) is explored. A risk-preference model, which relies on cumulative function using simulation, is introduced; it can rank, based on their determined values of expected utilities, pairs of alternative options representing risk factors or random variables (normalized lotteries) with equal expected values. Each variable, which can be differently distributed random variables that can possess various utility functions, is obtained by converting the outcomes of the lottery into its mean multiplied by a relative risk factor. With the existence of an expected utility model, the preference ordering is then converted into a risk-preference ordering over their risk factors. Simulation results show that the proposed model, in which ranking preferences is based on risk measures is an appropriate approximation model; it is flexible to solve large-scale decision problems that concern financial investment scenarios for risk-averse investors, where they can choose their preferences by focusing only on the risk factors.

4.1 Introduction

Many decision-making problems involve considerable uncertainty about the outcomes of alternative decisions. It is often necessary to evaluate risks for various alternatives in addition to their ordering based on preferences.

Risk and preference have been subjects of interest and study in many fields such as economics and psychology. In spite of the close connection between the two concepts, the modelling efforts have proceeded separately; theories and modelling approaches have been developed as independent concepts, while efforts to link these two concepts directly have been less common. As the result, two directions of research for decision- making problems under risk (uncertainty) are considered. The first direction is on measuring risk judgments for lotteries with no direct relationship with preference; risk is measured through direct judgments in the same way as psychological sensation such as brightness or loudness (Pollatesk and Tversky, 1970; Fishburn, 1982; Sarin, 1987). The second is on modelling choices or alternative preferences without invoking the notion of risk, for example, with the Expected Utility Theory (EUT) proposed by Von-Neumann and Morgenstern (VNM) (1944), the measure of risk aversion is simply based on the shape of the utility function and there is no exact relationship between risk and preference.

However, the theory of choice under uncertainty has been considered as one of the success issues of economic analysis, the VNM expected utility model and other generalizations, such as Prospect Theory (Kahneman and Tversky, 1979), Cumulative Prospect Theory (Tversky and Kahneman, 1992), and others, do not treat risk as a primitive. Since the rational behaviour under risk is to maximize an average expectation then the idea of risk is embedded in the idea of choice determined by the expected utility. Therefore, to study the effects of risk and uncertainty on preference in different areas, different theories for risky choices have been proposed. Furthermore, for most of the decision theories, the role of risk is usually considered implicitly in the preference model where risk is a related factor with preference. For example, Pratt (1964) proved that the riskiness of lotteries for an individual should be related to his utility model. Thus, the Expected Utility Model (EUM) provided by EUT can explain an explicit relationship between risks and preference and it would be possible to drive measures of risk alone from measures of preference.

In recent years, other researches (Dyer and Sarin, 1982; Sarin and Weber, 1993; Bell, 1995; Jia, 1995; Jia and Dyer, 1996; 2005) have attempted to link measures of risk directly to preference models and developed explicit forms of risk-value models (risk-return models). For example, Sarin and Weber (1993) proposed a risk-value model that unifies two streams of research: one in modelling preferences and the other in modelling risk judgments. In addition, it is shown that the mean variance frame work, proposed by Markowitz (1952; 1959), is a popular method for analyzing preferences, especially in the field of economics. However, despite that, regardless of their distributions, ranking alternatives are not always possible if only variances are considered (Sarin and Weber, 1993); empirical evidences, Table 3.3, from the implementation of the proposed algorithm, Algorithm 3.1, in the previous chapter, *Chapter 3*, has shown that, based on computed variances, ranking preference is practical. Hence, using the same methodology as described in *Chapter 3*, with the distinction that it is applied to normalize lotteries with the same expected values only, it is possible to introduce a new approach for analysing risk-preferences, based on the determined values of variances.

In this chapter, based on cumulative functions using simulation, a new modelling approach that links preference ordering of pairs of lotteries, with non-negative outcomes only, directly to a risk ordering, on risk factors, is introduced. Each risk factor, which is obtained from a multiplication decomposition of such lottery from one attribute structure in to two, the mean and a risk factor, is defined as the ratio of the lottery relative to its mean and represented by a normalized random variable/lottery. With the existence of expected utility theory, if the relative risk independence condition is satisfied, which assumes that the preference ordering over any pair of lotteries would not change if a constant is added (or subtracted) to the outcomes of these lotteries, then the preference ordering over any pair of such lotteries can be converted to a risk ordering on the risk factors obtained from such a decomposition structure. Hence, a risk-preference model that ranks pairs of uncertain lotteries, representing risk factors is proposed; if risk is involved then this factor is a measure of risk associated with the lottery relative to its expected value.

Furthermore, the approximation model, which is proposed under this approach, can be applied to differently distributed random variables, or lotteries representing financial investments, with a variety of utility functions; where investors can choose their preferences by focusing only on the risk factors that have the same expected values.

In the next section, the new proposed model on normalized lotteries, based on a decomposition structure, is introduced.

4.2 Model Based on Decomposition of Lotteries

The new modelling approach introduced in this chapter concerns normalized lotteries, with non-negative outcomes only, obtained by a multiplication decomposing of lotteries from one attribute structure into two attributes. In this section, the decomposition structure of a lottery to an alternative approach, multiplicative decomposition of the mean and a risk factor, is detailed below.

In decomposing lotteries from one attribute structure in to two, based on the fact that it is common practice to treat the non-negative outcomes of normalized lotteries, with equal expected values, as percentages based or return rates. Jia (1995) proposed several general measures of risk, the risk-value theory and the idea of risk-value trade offs which provides a link between a riskiness ordering and preference ordering, with the traditional expected utility model, and applied to some specific utility functions. Here, for each lottery, the same multiplication decomposition structure, its mean multiplied by a risk factor, is applied.

If a lottery, with non-negative outcomes, is defined as a random variable $X > 0$, then it can be written as $X = \overline{X} * \hat{X}$, $\hat{X} = \frac{X}{\overline{X}}$, where $\overline{X}$ is the mean and $\hat{X}$ is a relative risk factor for X. Hence, $\hat{X}$ is a normalized lottery defined as the ratio of the lottery X relative to its mean $\overline{X}$ with $E(\hat{X}) = 1$. Define P as the set of all of these lotteries (probability distributions); and $\hat{P} = \{\hat{X} : \hat{X} = \frac{X}{\overline{X}}, X \in P, X > 0\}$ as the set of all such normalized lotteries represented by the risk factors. Therefore, if a lottery X is decomposed as $X = \overline{X} * \hat{X}$ then it can be represented by a two-attribute structure of $(\overline{X}, \hat{X})$ on a product set of outcomes.

For this special case, the outcomes for the lottery on X_1 is fixed, which is the mean; thus the marginal distribution on X_1 is with one certain outcome $\overline{X}$. For the second attribute, the marginal distribution on X_2 is $\hat{X}$. Therefore, $(\overline{X},\hat{X})$ denotes a distribution that yields $\overline{X} \in X_1$ with probability 1 and $\hat{x} \in X_2$ with probability $\hat{X}$, where $\hat{x}$ is a realization of $\hat{X}$ with expected value $E(\hat{X}) = 1$. This implies that the evaluation of X can be separated to the evaluations of two attributes, $\overline{X}$ and $\hat{X}$.

Therefore, in order to consider an individual's preference ordering of lotteries in P together with the ordering for the lotteries in the risk factor set $\hat{P}$, it is needed to introduce the following, a binary preference relation denoted by $\underset{P}{\succ}$ and a binary risk relation denoted by $\underset{R}{\succ}$. Hence, to explain how a preference ordering for any pair of lotteries can be converted to a risk ordering over their risk factors, assume that $\underset{P}{\succ}$ denote a binary preference relation on the set P, analogous to the preference relation defined by expected utility theory and $\underset{R}{\succ}$ denotes a binary risk relation. These two relations can be explained as, for any pair of lotteries $X,Y \in P$, $X \underset{P}{\succ} Y$ means that the lottery X is preferable to the lottery Y, $Y \underset{R}{\succ} X$ means that the lottery Y is more risky than the lottery X. If $\underset{P}{\succ}$ satisfies the VNM expected utility axioms, then the preference relation can be represented by an expected utility model ($X \underset{P}{\succ} Y$ if and only if $E[u(X)] \geq E[u(Y)]$), E represents the expectation over the probability distribution of a lottery and u is unique up to a positive linear transformation.

Hence, by a simple transformation, an expected utility model defined on X, according to a multiplication relation, can be converted in to a two attribute expected utility model, the mean $\overline{X}$ and the risk factor $\hat{X}$, i.e., $E[u(X)] = E[u(\overline{X} * \hat{X})]$.

Thus, for any pair of lotteries $X,Y \in P$,

$$X \underset{P}{\succ} Y \text{ implies that } (\overline{X}, \hat{X}) \underset{P}{\succ} (\overline{Y}, \hat{Y}) \quad (4.1)$$

Where, $\overline{X}$ and $\overline{Y}$ are constants and $\hat{X}, \hat{Y} \in \hat{P}$ are normalized lotteries with the same expected values, if the relative risk independence condition holds (risk independence condition was studied earlier by many researchers, for example, Markowitz (1952); Edwards (1954); Kahneman and Tversky (1979); Jia (1995)), which assumes that for any pair of lotteries, an individual's preference ordering over these lotteries would not change if a constant is added (or subtracted) to the outcomes of these lotteries, then the only choice attribute of relevance for ranking preference in (4.1) is the risk factors and a riskier lottery on $\hat{P}$ would be less preferable and vice verse.

Thus, for this special case where the lotteries are decomposed in to their means multiplied by relative risk factors, the preference ordering over lotteries is the same as the ordering over their risk factors. Therefore, from this ordering relationship, it implies that the preference ordering over any pair of lotteries $X, Y \in P$ is equivalent to the ranking order over their risk factors represented by normalized lotteries $\hat{X}, \hat{Y} \in \hat{P}$.

4.2.1 Ranking model based on a multiplicative decomposition

Ranking alternatives to select the most desirable one is often a critical problem especially when risk attitudes crucially affect these preferences; decision makers therefore need to take risk aspects into account and attempt to conduct ranking strategies that are based on risk measurements. In the previous section, it has been shown that lotteries with non-negative outcomes only can alternatively be decomposed into the multiplication of the mean and the relative risk factors represented by normalized lotteries. In this section, in order to incorporate risk into ranking preferences, based on the multiplication decomposition structure of lotteries, a new risk-preference model is explained. It is based, as in *Chapter 3,* on both mean-variance analysis and cumulative distribution function, using simulation, with the distinction that it is applied to normalized lotteries representing risk factors.

4.2.2 Model for normalized lotteries

The new model described in this section is concerned with normalized lotteries, obtained by a multiplicative decomposition, with the same expected values but different risk measures represented by risk factors explained earlier. Since the developed model describes an application for financial investment then the risk aversion case is assumed where the riskiness ordering of lotteries in $\hat{P}$ should be the inverse of the preference ordering. Hence, from the relationship between risk and preference, the following can be introduced:

If $\underset{R}{\succ}$ denotes a binary risk relation on the set $\hat{P}$ (note that $\hat{P}$ is a subset of P) then for any pair of normalized lotteries $\hat{X}, \hat{Y} \in \hat{P}$, $\hat{X} \underset{P}{\succ} \hat{Y}$ ($\underset{P}{\succ}$ is a binary preference relation) means that the lottery $\hat{X}$ is preferable to the lottery $\hat{Y}$, Similarly $\hat{Y} \underset{R}{\succ} \hat{X}$ means that the lottery $\hat{Y}$ is riskier than the lottery $\hat{X}$. Thus, from equation (4.1), by assuming the relationship between $\underset{P}{\succ}$ and $\underset{R}{\succ}$ on the set $\hat{P}$, we can introduce the following definition

Definition 4.1: For any pair of normalized lotteries $\hat{X}, \hat{Y} \in \hat{P}$ with equal expected values, the risk relation and the preference relation satisfy a consistent condition such that $X \underset{P}{\succ} Y$ if and only if $\hat{Y} \underset{R}{\succ} \hat{X}$.

Thus, the following, which introduces a preference assumption that provides a fundamental condition called relative risk independence, is introduced. It is similar to the utility independence condition of the multi-attribute utility model described by Keeney and Raiffa (1976); it allows an expected utility model to be decomposed in terms of the mean and a relative risk factor.

Definition 4.2: For every binary relation $\underset{P}{\succ}$ on $\hat{P}$, $\hat{X} \in \hat{P}$ is relatively risk independent on $\overline{w} \in R^{+}$ means that (if for all $\hat{Y} \in \hat{P}$, there exist $\overline{w}_0 \in R^{+}$ such that $\overline{w}_0 * \hat{X} \underset{P}{\succ} \overline{w}_0 * \hat{Y}$ then $\overline{w} * \hat{X} \underset{P}{\succ} \overline{w} * \hat{Y}$ for all $\overline{w} \in R^{+}$), where R^{+} is the set of all positive real numbers.

Definition 4.2 explains relatively risk independence where $\overline{w}$ can be interpreted as an expected value and $\overline{w} * \hat{X}$ means that all outcomes of normalized lottery $\hat{X}$ is multiplied by a constant $\overline{w}$. Relative risk independence implies that, if two options have the same expected value, then the preference ordering over the two options will not change by increasing or decreasing the expected value by an equal amount.

Therefore, if $\underset{P}{\succ}$ relation satisfies the VNM expected utility axioms then the preference ordering can be represented by an expected utility model. Hence, based on the axioms of EUT, the consistent condition, Definition 3.1, makes it possible to drive a measure of risk on $\hat{P}$ and hence

Definition 4.3: Let $R(\hat{X})$ be a measure of risk defined on $\hat{P}$. It is defined as $R(\hat{X}) = -E[u(\hat{X})]$ (Jia, 1995).

Theorem 4.1: For any pair of normalized lotteries defined on $\hat{P}$, if both the consistency and the relatively risk independence conditions hold, then there exist a measure of risk $R: \hat{P} \rightarrow$ **Re** (**Re** is the set of real numbers), such that for any such pair $\hat{X}, \hat{Y} \in \hat{P}$, $\hat{X} \underset{R}{\succ} \hat{Y}$ if and only if $R(\hat{X}) \geq R(\hat{Y})$.

Proof: Suppose that for any pair of lotteries $\hat{X}, \hat{Y} \in \hat{P}$, if $\hat{X} \underset{R}{\succ} \hat{Y}$ then it is needed to show that $R(\hat{X}) \geq R(\hat{Y})$.

For any pair $\hat{X}, \hat{Y} \in \hat{P}$, if the consistency condition holds, then from Definition 4.1, $\hat{X} \underset{R}{\succ} \hat{Y}$ is equivalent with $\hat{Y} \underset{P}{\succ} \hat{X}$.

If $\hat{Y} \underset{P}{\succ} \hat{X}$ then from the expected utility theory, there exist $u: \hat{P} \rightarrow$ **Re**, such that $\hat{Y} \underset{P}{\succ} \hat{X}$ if and only if $E[u(\hat{Y})] \geq E[u(\hat{X})]$.

From definition 4.3, $R(\hat{X}) = -E[u(\hat{X})]$, $R(\hat{Y}) = -E[u(\hat{Y})]$, this implies that $E[u(\hat{Y})] \geq E[(u(\hat{X})]$. Or $-E[u(\hat{Y})] \leq -E[u(\hat{X})]$ implies that $R(\hat{Y}) < R(\hat{X})$ Or $R(\hat{X}) > R(\hat{Y})$.

Conversely, to show that if $R(\hat{X}) > R(\hat{Y})$ then $\hat{X} \underset{R}{\succ} \hat{Y}$,

If $R(\hat{X}) > R(\hat{Y})$ then $-E[u(\hat{X})] \geq -E[u(\hat{Y})]$ or $E[u(\hat{X})] \leq E[u(\hat{Y})]$

Hence $\hat{Y} \underset{P}{\succ} \hat{X}$. From the consistency condition, it follows that $\hat{X} \underset{R}{\succ} \hat{Y}$. ♦

Remark 4.1

In Theorem 4.1, a general measure of risk, relatively independent of its expected value, is derived. It does not impose any restriction on the distribution function for lotteries or the proposed utility function.

4.2.3 A risk-preference model based on both mean-variance analysis and cumulative function using simulation

In this section, the new proposed model, which concerns normalized lotteries only, is described; it is a risk-ranking model based on both mean-variance model and cumulative function. Using simulation, the inverse transformation method generates different values for the random variables, which represent normalized lotteries in $\hat{P}$, each defined as the ratio of the lottery in P relative to its mean, from the inverses of their cumulative functions. These values can then be used to represent the real-world random variables/ alternative options.

If a random variable $X > 0$, which represents a lottery in P (with non-negative outcomes), is considered, then it can be written as a decomposition structure as $X = \bar{x} * \hat{X}$, where $\bar{x}$ is the mean and $\hat{X}$ is defined as $\hat{X} = \frac{X}{\bar{x}}$ in $\hat{P}$. Hence, by transformation of variables, (Rohatgi and Saleh 2001), which determines the distribution function for a random variable $Y = g(X)$ from the distribution function of the random variable X with a probability density function $f(x)$, the distribution function for $\hat{X} \in \hat{P}$ can be determined from the distribution function of $X \in P$; this leads to the determination of the cumulative distribution function for $\hat{X}$. Therefore, it is possible to follow the same methodology explained in Chapter 3 and the approximation results from Levy and Markowitz (1979), Hlawitschka (1994), by assuming different utility functions such as power, linear power utility, quadratic or exponential utility function defined over real line.

If it is assumed that $(\hat{X}, \hat{Y}) \in \hat{P}$ is any pair of lotteries then the cumulative random variable $F_{\hat{X}}$ can be defined as a random variable associated with the distribution function $F_{\hat{X}}(x)$; and the cumulative random variable $F_{\hat{Y}}$ can be defined as a random variable associated with the distribution function $F_{\hat{Y}}(x)$. Consequently, for risk ranking preferences, the cumulative random variables $F_{\hat{X}}$ and $F_{\hat{Y}}$, rather than the lotteries can be used and risk ranking over normalized lotteries can be thought of as ranking over such cumulative random variables.

If preference ordering over cumulative random variables is consistent in the sense of VNM utility theory, then it can be represented by a utility function $u(\hat{x})$ so that $\int_{-\infty}^{\infty} u(\hat{x}) dF_{\hat{X}}(\hat{x}) \geq \int_{-\infty}^{\infty} u(\hat{x}) dF_{\hat{Y}}(\hat{x})$ if and only if $F_{\hat{X}}$ is ranked above $F_{\hat{Y}}$ in the preference ordering. For our proposed model, both $F_{\hat{X}}$ and $F_{\hat{Y}}$ are approximated random variables; each random variable is simulated, using inverse transform method, from different values generated from the inverses of the cumulative distribution functions.

Therefore, if the same approximation results for Levy and Markowitz (1979), Hlawitschka (1994), is followed then the approximated expected utility for such cumulative random variables, in terms of the two parameters the mean and the variance, can be determined, and hence the approximated $E[U(F_{\hat{X}})]$ for $F_{\hat{X}}$ can be expressed in terms of the two parameters, the mean and the variance. Thus,

$$E[U(F_{\hat{X}})] = U(\mu_{F_{\hat{X}}}) + \frac{U''(\mu_{F_{\hat{X}}})}{2} * \sigma^2{}_{F_{\hat{X}}} \tag{4.3}$$

Where $\mu_{F_{\hat{X}}}$ represents the mean and $\sigma^2{}_{F_{\hat{X}}}$ represents the variance for the cumulative variable $F_{\hat{X}}$, $U'' < 0$ (Pratt, 1964), is the second derivative for the utility function around the mean. Therefore, equation (4.3) is valid to determine the approximated value for the expected utility for such random variable using simulation.

Similarly, $E[U(F_{\hat{Y}})]$ for $F_{\hat{Y}}$ can be expressed in terms of the two parameters, the mean and the variance; the approximated value for expected utility then can be evaluated. Since the expected value for all normalized lotteries are equal then the preferred option is determined, which is the one with higher expected utility and less variance.

Remark 4.2: Dekking F. and Meester L. (2005), extended the transformation of variable for more general cases and obtained the following rule and call it Change-of-Units-Transformation, which concerns some special probability distribution functions related to our work. Thus, the following rules can be introduced.

Let X be a continuous random variable with distribution function $F_X(x)$ and probability density function $f_X(x)$. If the variable is changed to $Y = aX + b$ for real numbers $a > 0$ and b, then

$$F_Y(y) = F_X(\frac{y-b}{a}) \quad \text{and} \quad f_Y(y) = \frac{1}{a} f_X(\frac{y-b}{a}) \tag{4.4}$$

Thus, equation (4.4) can be applied to find the distribution function for $\hat{X}$ from the distribution function of X; it can be explained by giving the following

Example 4.1: Assume that a random variable X has the exponential probability distribution function with parameter λ, where $f(x,\lambda) = \lambda e^{-\lambda * x}$ for all $x > 0$ and $\lambda > 0$. The cumulative function $F(x)$ for X is determined as $F(x) = 1 - e^{-\lambda * x}$ for all $x > 0$.

If X is decomposed to $X = \bar{x} * \hat{X}$ then from (4.4), the probability distribution function $f(\hat{x})$ and the cumulative function $F(\hat{x})$ for $\hat{X}$ is determined as

$$f(\hat{x}) = \frac{1}{a} f_X(\frac{\hat{x}-s}{a}) \quad \text{and} \quad F(\hat{x}) = F_X(\frac{\hat{x}-s}{a}), \text{ where } a = \frac{1}{\bar{x}}, \; s = 0.$$

Thus, $f(\hat{x}) = \lambda * \bar{x} * e^{-\lambda * \bar{X} * \hat{X}}$ for all $\hat{x} > 0$, $F(\hat{x}) = 1 - e^{-\lambda * \bar{X} * \hat{X}}$ for all $\hat{x} > 0$ and $\lambda > 0$.

From Example 4.1, $\bar{x}$ is a constant and the distribution function for $\hat{X}$ is the same as that of X. Hence, to show that the preference ordering over lotteries, with non-negative outcomes only, can be converted into ordering over their risk factors, obtained from a multiplicative decomposition of a lottery into its mean and a relative risk factor, based on the above assumptions, the following theorem is introduced.

Theorem 4.2: Assume the existence of an expected utility model on a non-empty set of non-negative outcomes. For any pair of lotteries $(X,Y) \in P$, and its normalized pair $(\hat{X},\hat{Y}) \in \hat{P}$, obtained by a multiplicative decomposition of the mean and a relative risk factor, $X \underset{P}{\succ} Y$ if and only if $\hat{X} \underset{P}{\succ} \hat{Y}$.

Proof: If $X \underset{P}{\succ} Y$ then it is needed to show that $\hat{X} \underset{P}{\succ} \hat{Y}$.

Since each pair $X, Y \in P$ can be decomposed into a multiplicative form then X, Y can be written as $X = (\bar{x} * \hat{X}) = (\bar{x}, \hat{X})$, $Y = (\bar{y} * \hat{Y}) = (\bar{y}, \hat{Y})$ where $\bar{x}, \bar{y} > 0$ are their respective means and $\hat{X}, \hat{Y}$ are their risk factors, $\hat{X} = \dfrac{X}{\bar{x}}$, $\hat{Y} = \dfrac{Y}{\bar{y}}$.

If $\underset{P}{\succ}$ satisfies the VNM expected utility axioms then for each $X, Y \in P$, $X \underset{p}{\succ} Y$ if and only if $E[U(X)] \geq E[U(Y)]$. This implies that

$$X \underset{p}{\succ} Y \text{ if and only if } E[U(\bar{x}, \hat{X})] \geq E[U(\bar{y}, \hat{Y})] \tag{4.5}$$

Empirically, from equation (4.3), $E[U(X)] \geq E[U(Y)]$ is satisfied when the lottery X is with higher expected value and less variance than the lottery Y, this implies that equation (4.3) is satisfied if

$$(1)\ E(X) \geq E(Y), \text{ or } \bar{x} \geq \bar{y}, \text{ this implies that } \frac{1}{\bar{x}} \leq \frac{1}{\bar{y}}, \text{ and } \frac{1}{\bar{x}^2} \leq \frac{1}{\bar{y}^2} \tag{4.6}$$

$$(2)\ E(X - \bar{x})^2 \leq E(Y - \bar{y})^2 \tag{4.7}$$

From equations (4.6) and (4.7), these results are followed:

$\dfrac{1}{\bar{x}^2} * E(X - \bar{x})^2 \leq \dfrac{1}{\bar{y}^2} * E(Y - \bar{y})^2$, this implies that $\dfrac{E(X - \bar{x})^2}{\bar{x}^2} \leq \dfrac{E(Y - \bar{y})^2}{\bar{y}^2}$

$$\text{Or } E(\frac{X}{\bar{x}} - 1)^2 \leq E(\frac{Y}{\bar{y}} - 1)^2 \text{ implies that } E(\hat{X} - 1)^2 \leq E(\hat{Y} - 1)^2 \tag{4.8}$$

If $E(X) \geq E(Y)$ and $\bar{x}, \bar{y} > 0$ then $E(\dfrac{X}{\bar{x}}) \geq E(\dfrac{Y}{\bar{y}})$, this implies that:

$$E(\hat{X}) \geq E(\hat{Y}) \tag{4.9}$$

Therefore, from equations (4.8) and (4.9), it implies that $\hat{X} \underset{P}{\succ} \hat{Y}$.

Conversely, if $\hat{X} \underset{P}{\succ} \hat{Y}$ then $X \underset{P}{\succ} Y$. ♦

Remark 4.3: Theorem 4.2 explains that, if normalized lotteries are considered then preference ordering of any pair of such lotteries is equivalent to the risk ordering for their relative risk factors.

Therefore, if normalized lotteries are considered then a new approach is proposed; it is based on both the mean-variance analysis, consistent with the VNM axioms, and the cumulative distribution function. With this model, which is a modification for MVM, the requirement of having to define random variables from the distribution functions in $\hat{P}$ can be replaced by defining random variables from the cumulative distribution functions which can be obtained from the distribution function of the lottery in the probability set defined by P.

In the next section, an algorithm is proposed to describe the new risk ranking model proposed for normalized lotteries with non-negative outcomes only.

4.3 Algorithm and Simulation Results

The algorithm explained in this section describes the new proposed risk-ranking model that concerns normalized lotteries in $\hat{P}$ with non-negative outcomes only. It determines the distribution functions for lotteries in $\hat{P}$ from the distribution functions for lotteries in P. Hence, the simulation procedure for generating random values is carried out in two steps. First, it generates values from the cumulative function $F_X(x)$ for the lottery X in P, and then computes the mean $\overline{X}$ for these generated values. Second, by generating values for $\hat{X}$ from cumulative distribution function $F_{\hat{X}}(\hat{x})$, which is determined from the probability function for X in P, it depends on the computed value of $\overline{X}$ from the first step. Finally, from (4.3), the algorithm computes the expected utility for both randomly generated variable

$F_{\hat{X}}$ and $F_{\hat{Y}}$. The preferred lottery then would be the one with higher expected value, which is one with less variance as the expected values for each pair approximately are equal. The main steps are detailed below.

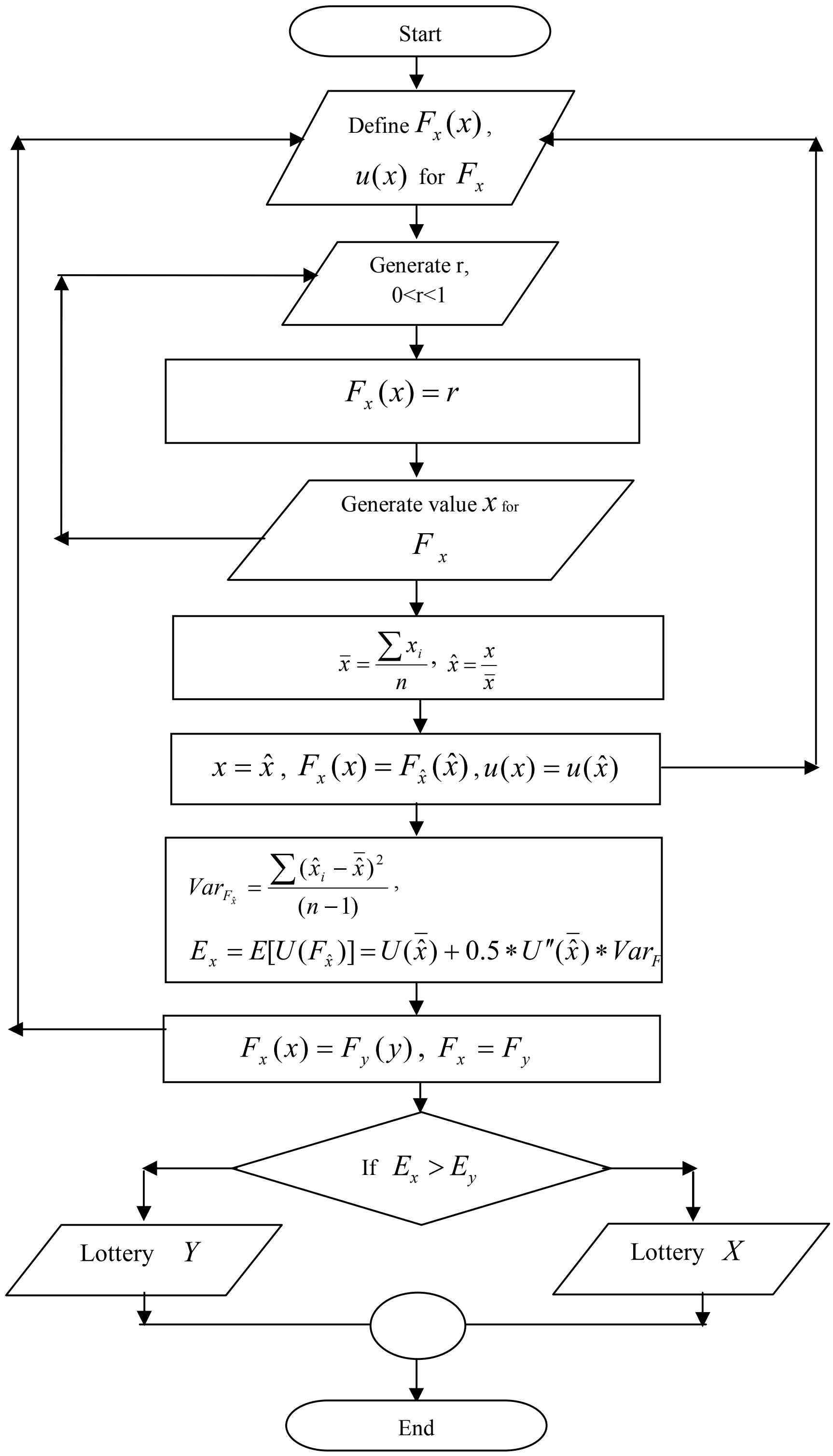
4.3.1 Algorithm 4.1

The main steps for the algorithm that describes the new proposed risk-ranking model can be summarized below:

1. Define the cumulative distribution function $F_X(x)$ for the random variable F_X which possesses a utility function $u(x)$;
2. Generate random numbers r, $0 < r < 1$ from uniform distribution function;
3. Equate $F_X(x)$ to r;
4. Generate the first value x for F_X from the inverse function F^{-1}, $x = F^{-1}(r)$;
5. Repeat 2-4 to generate different values for F_X;
6. Find the mean $\bar{x}$ for the generated values x for F_X;
7. Repeat 2-4 to generate new values x for F_X, then compute $\hat{x} = \dfrac{x}{\bar{x}}$
8. Repeat 7 to generate different values for $F_{\hat{X}}$ from cumulative function $F_{\hat{X}}(\hat{x})$;
9. Determine the variance for the generated values representing the random variable $F_{\hat{X}}$, then compute the expected utility for $F_{\hat{X}}$ from (4.3), considering a specific utility function ;
10. Repeat 1-9, by considering the random variable $F_{\hat{Y}}$ with cumulative function $F_{\hat{Y}}(\hat{x})$ and the same utility function as for $F_{\hat{X}}(\hat{x})$;
11. The preferred lottery is the one with higher expected value and least variance for all generated values.

The main steps of the algorithm can be illustrated by the following flowchart.

Flow Chart 4.1: Algorithm 4.1 flow chart



4.3.2 Implementation and simulation results

For the implementation of Algorithm 4.1, the simulation procedure was practised using MATLAB; a simulation routine generated different uniform random numbers representing an alternative option. Each simulation run consisted of a specified number of replications denoted by n. When $n = 10000$ was experienced the running time was found to be efficient; several other cases were run using $n = 100000$ and the results were almost the same; while larger n led to overtime running. Using these random numbers, different values for the random variable from its inverse cumulative distribution functions are generated to represent the simulated random variables; each is representing an alternative option. Hence, the mean, the variance and the expected utility were determined according to some specific utility function and for some specific values of parameters, which was given by the user.

The simulation procedure to generate other alternative was similar, except that different parameter values were given; ranking preference for each pair of alternatives was determined from the simulation results. The most preferred option, generated random variable, was the one with higher expected utility and less variance as the expected values for the simulated random variables, approximately, equal one.

Tables (4.1-4.4) display simulation results for implementing the proposed algorithm, run for various pairs of lotteries with different parameters and utility functions in addition to the results obtained for lotteries in P. However, with this model, the ranking preference will not change if an amount is added (or subtracted) to the mean of each pair, it is independent on the measure of the anticipated mean and only the measures that compute variances and the expected utilities are considered.

As in Table 3.1, Table 4.1 represents simulation results for generating a pair of normally distributed lotteries, $\hat{X}, \hat{Y} \in \hat{P}$ with different means and equal variances and the utility function is considered as a power function; it is defined as

$$u(x) = \frac{x^{(1-\theta)}}{(1-\theta)}, \quad 0 < x \le 1 \text{ and } 0 < \theta < 1 \text{ with } u''(x) = -\theta * x^{(-\theta-1)} < 0$$

The number of replications considered is $n = 10000$ and the parameter for power function is $\theta = 0.5$. The random variable X is assumed to be normally distributed with mean equals one and variance equals one, i.e. $X \sim N(1,1)$.

The new algorithm first, generates random values for $X \in P$, where $X \sim N(1,1)$, then determines the mean $\bar{x}$ for the generated values. Second, it determines the cumulative function for $\hat{X} \in \hat{P}$ using equation (4.4), and then generates random values for $\hat{X} \in \hat{P}$.

If a random variable $X \sim N(\mu,\sigma^2)$, this implies that the random variable $C * X$ is normally distributed, i.e. $C * X \sim N(C * \mu, C^2 * \sigma^2)$, for any constant $C \neq 0$.

If $C = \frac{1}{\bar{x}}$ then $\frac{X}{\bar{x}} \sim N(\frac{\mu}{\bar{x}}, \frac{\sigma^2}{\bar{x}^2})$. Therefore, random values for $\hat{X}$ are generated from the inverse of the normal cumulative function $F(\hat{x})$ with mean $\frac{1}{\bar{x}}$ and variance $\frac{1}{\bar{x}^2}$, then, for these generated values, the mean, the variance and the expected utility is computed.

The same procedure is practised to generate values for $\hat{Y}$, then compute the mean, the variance and the expected utility for generated values, by assuming a normally distributed random variable $Y \sim N(2,1)$. Finally, from the implementation of the algorithm, the second lottery is found to be more preferred as its generated values have the higher expected utility with less variance. It is noted that, for each lottery, the computed mean for the simulated random variable is approximately one; therefore, the selection of the most preferred lottery is independent on the mean measures.

Moreover, Table 4.1 presents simulation results for a pair of lotteries in P (non-normalized lotteries) with the same specifications as for that one in $\hat{P}$. It is noted that, from the simulation results for lotteries in $\hat{P}$ as well as in P, the preference ranking for each pair does not change. The determined expected utility for the second lottery, for both normalized and non-normalized lotteries, is higher than that for the first lottery, and therefore, the second lottery is preferred more.

In Table 4.2, it is shown the same results as for Table 4.1, implemented for normal random variables with equal means, but different variances, where $X \sim N(2,1)$, $Y \sim N(2,5)$ the parameter for power utility function is assumed to be $\theta = 0.9$, and the same results are obtained, the most preferred option is the first one, as it has the higher expected utility and less variance. The simulation results, for normalized and non-normalized lotteries, show that the expected utility for the first lottery is higher than that for the second one; hence the first lottery is the one which is preferred more.

In Table 4.3, a similar pair, as in Table 4.2, is considered; the random variables are normally distributed with the same mean and different variances with the distinction that the utility function considered is a logarithmic function. The simulation results for normal random variables $X \sim N(2,1)$ and $Y \sim N(2,2)$, and a logarithmic utility function show that the first lottery that has the higher expected value and less variance is preferred more.

Table 4.4 assumes non-normal random variables, an exponential random variable with parameter λ, and a power utility function with $\theta = 0.5$ are considered. For the first lottery $\lambda = 0.5$, for the second lottery $\lambda = 1.5$. Likewise, the simulated results show that the second lottery is preferred more to the first one.

The same algorithm was applied to other pairs of lotteries, differently distributed random variables such as lognormal, logarithm and power, where various utility functions were considered using different parameter values. The simulation runs resulted out were almost the same; for each pair, the most preferred lottery was the one with the higher expected value and less variance. Therefore, for ranking preferences, the proposed algorithm can be implemented by only focusing on risk factors represented by lotteries in $\hat{P}$, with different probability distributions. It is not necessary for the utility function to be defined only as a power function; it can consider other functions such as exponential, linear exponential or logarithmic function, or the random variables need not only to be normally distributed; they can be distributed as lognormal random variables, or other distributions according to the application field of this model.

Finally, it is concluded that, the determination of the most preferred lottery can be obtained directly from the implementation of the algorithm; it determines, for each uncertain alternative with the same mean, the expected utility value from the determined variance to be used as the basis for ranking preferences.

Remark 4.4

The simulation algorithm, Algorithm 4.1, in this chapter provides a novel approach, which is a flexible and practical tool, for modelling risk preferences with many desirable results. In contrast to other approaches, for example, Sarin and Weber (1993) ensure that: the variance, which is the measure of risk, cannot be used as the basis for ranking alternatives, regardless of their distributions, simulation results in this chapter show empirical evidences which ensure that regardless of their distribution, ranking preferences are possible even if only risk measures are considered. It is illustrated that this new approximation approach, which relies on cumulative functions, is flexible to handle large-scale decision problems that concern applications in financial economics, where risk is considered as an important element, this supports findings for other studies (Bell, 1988; Jia, 1995). In addition, it is expected to be flexible for modelling preferences in other fields of applications; this can be considered for further research studies. However, this modelling approach, as for other risk measures, for example Jia (1995), is based on the concept of decomposition of a lottery in to a multiplication of its mean and a risk factor, which is defined as the relative ratio of the lottery to its mean; it provides a link between a riskiness ordering and a preference ordering; but it is typically related only to the logarithmic functions and power functions, while this new approach, using simulation, can be related to random variables with various distribution functions and different utility functions. Therefore, the new approach for ranking risk-preferences can overcomes the limitations of other existing approaches; it is flexible to handle large-scale decision problems.

The following tables, Table 4.1-4.4, represent the obtained simulation results for the simulation runs, for both normalized lotteries in $\hat{P}$ and lotteries in P, with specified number of replications and for different values of parameters.

Table 4.1: Normal random variables X, Y with utility function defined as a power function

Parameter Values $n = 10000, \theta = 0.5$	**Simulation Results for Lotteries in $\hat{P}$**	**Simulation Results for Lotteries in P**
$\mu_X = 1.0$ $Var_X = 1.0$	$\mu_{XSimulated} = 1.000$ $Var_{XSimulated} = 0.991$ $E[U(X_{Simulated})] = 1.748$	$\mu_{XSimulated} = 1.008$ $Var_{XSimulated} = 0.994$ $E[U(X_{Simulated})] = 1.762$
$\mu_Y = 2.0$ $Var_Y = 1.0$	$\mu_{YSimulated} = 0.999$ $Var_{YSimulated} = 0.247$ $E[U(Y_{Simulated})] = 1.996$	$\mu_{YSimulated} = 2.004$ $Var_{XSimulated} = 1.003$ $E[U(Y_{Simulated})] = 2.243$

Table 4.2: **Normal random variables X, Y with utility function defined as a power function**

Parameter Values $n = 10000, \theta = 0.9$	Simulation Results for Lotteries in $\hat{P}$	Simulation Results for Lotteries in P
$\mu_X = 2.0$ $Var_X = 1.0$	$\mu_{XSimulated} = 1.000$ $Var_{XSimulated} = 0.246$ $E[U(X_{Simulated})] = 9.967$	$\mu_{XSimulated} = 2.013$ $Var_{XSimulated} = 1.003$ $E[U(X_{Simulated})] = 10.604$
$\mu_Y = 2.0$ $Var_Y = 5.0$	$\mu_{YSimulated} = 0.999$ $Var_{YSimulated} = 1.212$ $E[U(Y_{Simulated})] = 9.304$	$\mu_{YSimulated} = 1.983$ $Var_{XSimulated} = 5.007$ $E[U(Y_{Simulated})] = 8.805$

Table 4.3: **Normal random variables X, Y with utility function defined as a logarithmic function**

Parameter Values $n = 10000$	Simulation Results for Lotteries in $\hat{P}$	Simulation Results for Lotteries in P
$\mu_X = 2.0$ $Var_X = 1.0$	$\mu_{XSimulated} = 1.000$ $Var_{XSimulated} = 0.249$ $E[U(X_{Simulated})] = -0.04$	$\mu_{XSimulated} = 1.994$ $Var_{XSimulated} = 1.005$ $E[U(X_{Simulated})] = -1.317$
$\mu_Y = 2.0$ $Var_Y = 2.0$	$\mu_{YSimulated} = 0.999$ $Var_{YSimulated} = 0.512$ $E[U(Y_{Simulated})] = -0.12$	$\mu_{YSimulated} = 2.023$ $Var_{XSimulated} = 2.011$ $E[U(Y_{Simulated})] = -7.571$

Table 4.4: Exponential random variables X, Y with utility function defined as a power function

Parameter Values $n = 10000, \theta = 0.5$	**Simulation Results for Lotteries in** $\hat{P}$	**Simulation Results for Lotteries in** P
$\lambda = 0.5$	$\mu_{XSimulated} = 1.006$ $Var_{XSimulated} = 1.028$ $E[U(X_{Simulated})] = 0.744$	$\mu_{XSimulated} = 0.504$ $Var_{XSimulated} = 0.498$ $E[U(X_{Simulated})] = 0.332$
$\lambda = 1.5$	$\mu_{YSimulated} = 1.000$ $Var_{YSimulated} = 0.977$ $E[U(Y_{Simulated})] = 0.766$	$\mu_{YSimulated} = 1.986$ $Var_{XSimulated} = 1.975$ $E[U(Y_{Simulated})] = 1.637$

4.4 Summary

In this chapter, a novel approach based on another reformulation for MVM, which concerns risk measurements, was explored. The new risk-preference model, which is based on cumulative functions using simulation, was developed to rank pairs of normalized lotteries, random variables with non-negative outcomes only with the same expected values. Each variable, which represented a risk factor, was obtained from a multiplicative decomposition of each lottery into its mean multiplied by a relative risk factor. With the existence of an expected utility model and the realization of the relative risk independence condition, the preference ordering for each pair of these lotteries converted to a risk-preference ordering over the pair of their risk factors. Using simulation, a new algorithm was introduced; it represented uncertain alternatives (lotteries) by generating different random values from the inverses of the cumulative functions of random variables. The variance and the expected utility, for each of the simulated variables, were computed and the most preferred lottery was identified as having the higher expected value and less variance.

Therefore, the fundamental assumption for this modelling approach, which provided a preference basis, is the decomposition of the lotteries into a new separated form of equal means and different variances; each is represented by a risk measure. The relative risk independence condition, which assumes that the preference ordering for any two normalized lotteries, with the same expected values, would not change if any constant is added or subtracted to the outcomes of these lotteries, led to introduce a new MVM, which deals with risk preferences in decision making problems. However, the applications to this framework have shown that this approximation model is flexible for modelling risk preferences; it can deal with applications in financial investments, where risk is considered as an important element, the preference ordering would then be determined by just focusing on their risk factors. Moreover, it can be applied to normalized lotteries only, regardless of their distributions with various utility functions, based on the determined values for variances, where there is no need to consider the determination of means as they possess the same expected values. Finally, from the application of this approach it is concluded that it is usable to handle large-scale decision problems, which consider risk as an important factor, especially those encountered in financial problems. In addition, it is expected that this approach can be applicable to deal with decision problems in other fields of applications; this can be considered for future researches.

CHAPTER FIVE: A NEW AHP-BASED MODEL WITH A SPECIFIED LEVEL OF UNCERTAINTY

In this research study, for solving decision problems, especially decisions concern business projects for financial investments, preference methods have been proposed to evaluate pairs of alternative options to state that one option is more preferred than the other. However, the way in which the preference information, which was based on weight scales represented by the determined values for the expected utility, was processed is rather objective; the subjective judgments for the decision makers, which may significantly effect the decision process, have often been ignored. To deal with the main shortcomings of such methods, this chapter introduces a new methodology for ranking pairs of alternative options based on the Analytic Hierarchy Process (AHP). First, to overcome the main deficiency of AHP, this chapter introduces a new algorithm, using simulation; it allows limiting the uncertainty, which is inherent with AHP, to an accepted level. Hence, the study introduces a novel approach that combines both modified methods, the preference-based and AHP, to propose a new ranking strategy. The model described under this approach has the capacity to handle a great number of criteria; it allows incorporating judgmental perceptions, for experienced professionals, in to the decision process. Meantime, the integrated measurement obtained by this combination, which is based on more than one weighted scale from different sources, yields more accurate results in terms of identifying the best option. In addition, this approach provides an efficient approximation tool that can guide decision maker to make logical decisions to solve large-scale problems especially those concern financial economics. Following this introduction, the new algorithm and the procedure for the proposed model are explained.

5.1 Introduction

In decision making process, AHP (Saaty, 1980; Malhotra, 2001; Li and Li, 2009) has seen as a useful tool and a flexible model that deals with the qualitative variables and allows decision makers to make decisions by personal judgments in a logical way. AHP methodology, which is explained earlier, *Chapter 2*, is based on a pair-wise comparison procedure, which compares criteria, or alternatives, with respect to a criterion to establish the preference matrices. Hence, the nine-point scale, from 1-9, is used; each number represents one's subjective judgments to the decision process. For each level, with respect to the main goal, pair-wise comparisons are performed and then, for each pair of uncertain alternatives, the most preferred option is identified.

Despite its popularity, the AHP method does not take into account the uncertainty associated with the mapping of one's subjective judgment to a number. Consequently, this has often led the method to be criticised for not adequately handle the inherent uncertainty results from the mapping process. Thus, though AHP is advantageous in many aspects, the problem of uncertainty has remained to be a bottleneck of AHP. For this purpose, to study the uncertainty within AHP and then analyse its effects, various methods and simulation approaches has been developed (Paulson and Zahir, 1995; Hauser and Tadikamalla, 1996; Beynon, 2002; Wu, 2007; Lin et al., 2008).

In this chapter, in order to be able to propose an integrated approach for ranking preferences based on AHP, resolving the main deficiency of AHP, which is its inherent uncertainty, is considered. It, first, explains the main concept of uncertainty within AHP and then introduces a new algorithm that describes an approximation method, using simulation. The main contribution of this algorithm is in modifying AHP to be with a specified level of uncertainty. These are detailed below and then later in this chapter, based on this algorithm, the integrated model is introduced.

5.2 Uncertainty within AHP

In a decision process using AHP, to drive ratio scales of relative importance for decision criteria and options at each level in the hierarchy, decision makers need to set up preference matrices to do pair-wise comparisons. However, in most situations, as it is not realistic that the decision maker has either complete information regarding all factors of the decision making problem or full understanding of the problem to give the right judgment, the pair-wise comparison contain a degree of uncertainty which might result from a number of factors including the subjective state of the decision maker and insufficient sources of information. Therefore, when the decision maker has even doubt about his judgment then his pair-wise comparison matrix contains a degree of uncertainty that will certainly effect the final decision. Therefore the weakness of AHP is in assessing the relative importance weights of various criteria affecting the decision process; this mainly results from the uncertainty of using Saaty's nine-point scale to reflect the subjective opinion of decision makers in the judgmental matrix representing the relative importance relation ship among the various criteria under consideration. However, judgmental uncertainty is quite different from that of inconsistency, which was explained earlier, *Chapter 2*, which means contradictory in preferences, and can easily be measured; it is believed that a decision maker may express highly inconsistent preferences with a very low of uncertainty (Paulson and Zahir, 1995).

The concept of judgmental uncertainty, within AHP, was first defined by Saaty (1978), then later, the effect of uncertainty in judgments on the decision process, has been studied and investigated by many others (Zahedi, 1986; Saaty and Vargas, 1987; Millet and Wedley, 2002; Wu, 2007). These studies mostly deal with uncertainty to analyse its effect on the problem of rank reversal. Consequently, in AHP, the problem of uncertainty, which reduces the confidence of the users on the final results, always appears; this has often led the method to be criticised, for example studies by Dyer (1990); Belton and Gear (1993); Wang and Elhag (2006). Despite that, others (Saaty and Vargas, 1987; Saaty, 1990; Saaty and Vargas, 2006) responded to the criticism showing that AHP principles and scales have a solid theoretical and practical basis; this deficiency has remained the bottleneck of the AHP method.

However, in this study, where only pairs of alternative options are considered in which uncertainty leaves the rank of alternatives unchanged, to analyse uncertainty in AHP, this chapter emphasises on the importance of the subjective judgments of decision makers and their mapping numbers in the decision process. It introduces, using simulation, a new algorithm that describes an approximation approach to restrict the uncertainty within AHP, which results from the difficulty of composing the right preference matrix to represent decision maker's judgments. The simulation procedure described by this approach allows the uncertainty to be limited to an acceptable level; this can be identified by the decision maker at the beginning of the simulation process. It accepts only matrices with variances, of their corresponding Eigen-vectors, that do not exceed that limit. Meantime, the procedure allows verifying rank uncertainty, which is described by Saaty (1978) and supported by other studies, for example, Wu (2007); it ensures that when the uncertainty increases its rank uncertainty becomes higher. These are detailed below.

5.3 The Proposed Algorithm

In this section, an algorithm that deals with the weakness of uncertainty, which is occurred as the result of mapping process, in AHP is proposed. It introduces a new simulation procedure that emphasizes on controlling the uncertainty, which is resulted from the judgmental diversities for the experts represented by variances, and restricting it to a certain limit. For each preference matrix, it is assumed that the average of variances, for the Eigen-vector, is denoted by σ; the algorithm bounds this average to an accepted limit, specified by the decision maker. This allows, for each preference matrix, the variety in expert's opinions, which are mapped in to numbers 1-9, not to exceed that limit. The main procedure for the proposed algorithm is explained.

5.3.1 Algorithm 5.1

This algorithm presents a novel simulation technique that restricts the uncertainty within AHP, for each preference judgmental matrix. In this algorithm, it is assumed that the judgmental uncertainty for a preference matrix, which is denoted by σ, represents the consensus of variances within the same matrix. This value is identified by the decision maker at the beginning of the process of applying AHP and assumed to be the boundary for the uncertainty within the same matrix. Using simulation, it generates different normal random preference matrices. For each, the Eigen-vector and its associated variance are determined. Hence, a matrix can be identified and accepted as the desired preference matrix, with a variance of its associated Eigen-vector does not exceeds the boundary level σ, that satisfies the condition of having uncertainty bounded to the required level. Otherwise, the procedure of the algorithm will be repeated until the required matrix, which will possess a specified level of uncertainty, is obtained.

The proposed algorithm, for bounding the uncertainty within AHP, to some value σ, is explained in the following steps:

Step 1: Generate a reciprocal matrix A from the judgments of decision makers, $A = a_{ij}$ where a_{ij} is a uniform random values lies between 1-9 and the reciprocal a_{ji} is set equal to $1/a_{ji}$.

Step 2: Generate a set of N random matrices $[A^1, A^2, ..., A^N]$, assuming that elements of each A^K is normally distributed random variant with mean equal μ_{ij} and variance is σ_{ij}. For each A^K, elements $a_{ij}{}^K$ are generated from Box and Muller formula (Box and Muller, 1958), where $a_{ij}{}^K$ are generated as follows:

I. If $a_{ij} \geq 1$, means that each element of A is greater than or equal one, then $a_{ij}^{K} = \sqrt{-2 * \ln(u_1)} * \cos(2\pi * u_2) * \sigma_{ij} + a_{ij}$, where $u_1, u_2 \approx (0,1)$, random number generated from the uniform distribution function;

II. If $a_{ij} < 1$ then $a_{ij}^{K} = \dfrac{1}{2 - a_{ij}}$;

III. Else $a_{ij}^{K} = {}^{1}\!/_{a_{ij}}$, for all $i, j = 1,2,...,n, k = 1, N$.

Step 3: Calculate W^{K}, the principal Eigen-vector for the reciprocal matrix for each A^{K}.

Step 4: Calculate the mean w_i and the variance σ_i for each principal Eigen-vector W^{K}.

Step 5: If $\sigma_i \leq \sigma$ (the limit value for uncertainty) accept the random matrix as the preference matrix, otherwise step 2 is repeated.

In this algorithm, Step 1 generates the reciprocal matrix A, where a_{ij}, elements of A, are obtained from the decision maker's opinion mapped into Saaty's nine-point scale, 1-9, and when $i = j$, which means that the preference, or the importance of one criterion, or alternative to itself is the same, then $a_{ij} = 1$. When $i > j$, then the reciprocal element a_{ji} are defined, $a_{ji} = 1 / a_{ij}$.

In Step 2, N random matrices $[A^{1}, A^{2}, ..., A^{N}]$ are generated and elements $a_{ij}{}^{K}$, for each matrix A^{K}, are generated from Box and Muller formula.

Then, in Step 3, for each of the generated normal reciprocal random matrix A^{K} the principle Eigen-vector is determined, from the principle of linear algebra, the Eigen-value technique. The variance, for each of Eigen-vectors, is then determined from Step 4.

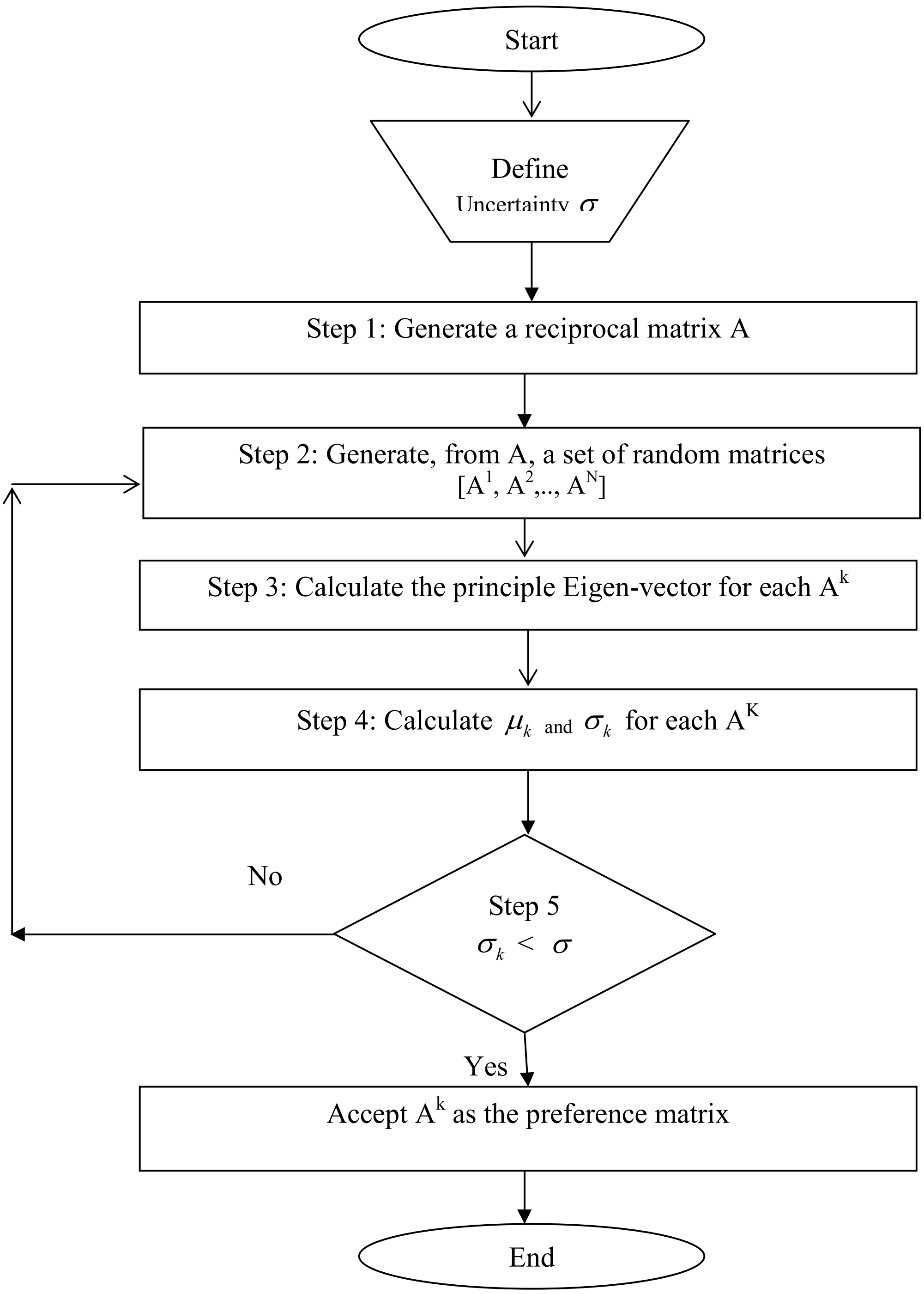
Step 5 compares the computed variance with the boundary value of the required variance, if the variance of the Eigen-vector for a generated matrix does not exceed this boundary then this matrix can be accepted as a preference matrix, which would be used for processing the AHP technique, otherwise the whole procedure will start again from Step 2 until the required preference matrix is found.

Remark 5.1

The main contribution of this algorithm, Algorithm 5.1, is in modifying AHP to correspond with the problem of uncertainty inherent within the method as the result of mapping judgments into real numbers. Based on simulation, this new algorithm introduces a novel approximation method, which is based on the fact that it is applied to pairs of options that keep ranking reversal unchanged, that allows limiting uncertainty to an acceptable level. Therefore, this simulation approach provides a simple approximation tool that can easily deal with uncertainty based on a limit identified by the user of this algorithm; the practical consequence, especially for researchers using AHP as a tool for pair-wise comparisons, is the specification of the boundary for the accepted uncertainty within preference matrices in decision hierarchies, it helps decision maker to be aware of the level of diversity in judgments before starting the decision process. Therefore, this practical algorithm can handle uncertainty in an easier way than others, for example, simulation approach by Paulson and Zahir (1993) mainly deals with uncertainty and its effect on rank reversal while the one by Wu (2007) helps to reduce uncertainty to some extent.

Flow Chart 5.1 illustrates the main steps for this algorithm.

Flow chart 5.1: Algorithm 5.1 Flow Chart



5.3.2 Simulation results

When Algorithm 5.1 is applied to a preference matrix, obtained from the preference judgments of the decision makers, each simulation run consists of a certain number of replications n specified at the beginning of the program with identifying an accepted limit of uncertainty defined by σ. Using simulation routines, pairs of uniform random numbers are generated to be used in generating different normal random matrices. For each of these matrices, the Eigen-value technique, from the principle of linear algebra, is employed to calculate the principle Eigen-vector, and then its variance is calculated. Hence, the matrix that has the variance of its Eigen-vector does not exceed the specified boundary σ is accepted as a preference matrix; it is used for processing the AHP technique. This algorithm was applied to various preference matrices representing different scenarios to identify a normalized preference matrix that is with an acceptable level of uncertainty; each implementation to this algorithm gave satisfactory results. One of such examples and its outputs is given below.

Example 5.1:

Consider a scenario where it is assumed that a decision maker has a preference matrix A; the elements of the matrix are defined as:

$$A = \begin{pmatrix} 1 & 5 & 7 & 9 \\ 1/5 & 1 & 1/5 & 7 \\ 1/7 & 5 & 1 & 1/3 \\ 1/9 & 1/7 & 3 & 1 \end{pmatrix}$$

It is assumed that the boundary of uncertainty is $\sigma = 0.014$.

To run this algorithm, if the number of replications is chosen to be $n = 100$, or smaller then the whole process is found to run faster while larger n causes delay in processing.

Therefore, when Algorithm 5.1 was applied to Example 5.1, the following results were obtained:

The number of replications $n = 100$

The limit for the variance $\sigma = 0.014$

The preference matrix $A(5x5)$

1.0000	0.3333	0.5000	0.5000	7.0000
3.0000	1.0000	1.0000	2.0000	9.0000
2.0000	1.0000	1.0000	1.0000	7.0000
2.0000	0.5000	1.0000	1.0000	7.0000
0.1429	0.1111	0.1429	0.1429	1.0000

The generated Normal Random Matrix M

0.9795	0.3333	0.5000	0.5000	6.9890
3.0106	1.0014	0.9768	2.0060	8.9961
1.9936	1.0035	1.0210	0.9784	7.0064
1.9894	0.5000	1.0050	0.9958	6.9906
0.1429	0.1111	0.1429	0.1429	0.9926

The Normalized Matrix for M

0.1207	0.1130	0.1372	0.1082	0.2256
0.3709	0.3395	0.2679	0.4339	0.2904
0.2456	0.3402	0.2801	0.2116	0.2262
0.2451	0.1695	0.2757	0.2154	0.2257
0.0176	0.0377	0.0392	0.0309	0.0320

The Eigen-vector for M is

0.1409 0.3406 0.2608 0.2263 0.0315

The Average Variance for Eigen-vector= 0.0141

This average is not accepted; take a new Random Normal Matrix and a new Eigen Vector;

The generated Normal Random Matrix M

1.0049	0.3333	0.5000	0.5000	7.0158
3.0161	1.0070	1.0106	1.9938	9.0005
1.9921	1.0098	1.0036	0.9764	6.9993
1.9712	0.5000	0.9735	0.9922	7.0075
0.1429	0.1111	0.1429	0.1429	1.0091

The Normalized Matrix for M

0.1236	0.1126	0.1377	0.1086	0.2261
0.3711	0.3400	0.2784	0.4329	0.2900
0.2451	0.3410	0.2764	0.2120	0.2255
0.2425	0.1688	0.2681	0.2154	0.2258
0.0176	0.0375	0.0393	0.0310	0.0325

The Eigen-vector for M

0.1417 0.3425 0.2600 0.2242 0.0316

The Average Variance for Eigen-vector= 0.0141

This average is not accepted; take another Random Normal Matrix and a new Eigen-vector;

The generated Normal Random Matrix M

0.9894	0.3333	0.5000	0.5000	7.0315
3.0115	0.9985	0.9939	1.9924	9.0002
2.0078	1.0023	0.9994	0.9938	6.9702
1.9966	0.5000	1.0006	1.0010	6.9958
0.1429	0.1111	0.1429	0.1429	1.0311

The Normalized Matrix for M

0.1214	0.1132	0.1375	0.1080	0.2266
0.3696	0.3390	0.2733	0.4303	0.2901
0.2464	0.3403	0.2748	0.2146	0.2246
0.2450	0.1698	0.2751	0.2162	0.2255
0.0175	0.0377	0.0393	0.0309	0.0332

The Eigen-vector for M is

0.1413 0.3405 0.2602 0.2263 0.0317

The Average Variance for Eigen-vector= 0.0139

This variance is accepted; accept the matrix M as the Preference judgmental Matrix with a level of uncertainty, which does not exceed 0.014.

The obtained simulation results show that, for each run, first, a normal random matrix M, from the original preference matrix A, is generated. Hence, it is converted in to a normalize matrix where the variance of the principle Eigen-vector is determined and then compared with the assumed value for σ. If the computed value, which is resulted from implementing the algorithm, is determined to be less than the value of σ then the normal matrix is accepted to be with uncertainty not exceeds the required limit. Otherwise, the normalize matrix is needed to be replaced by another one and then re-start.

Remark 5.2

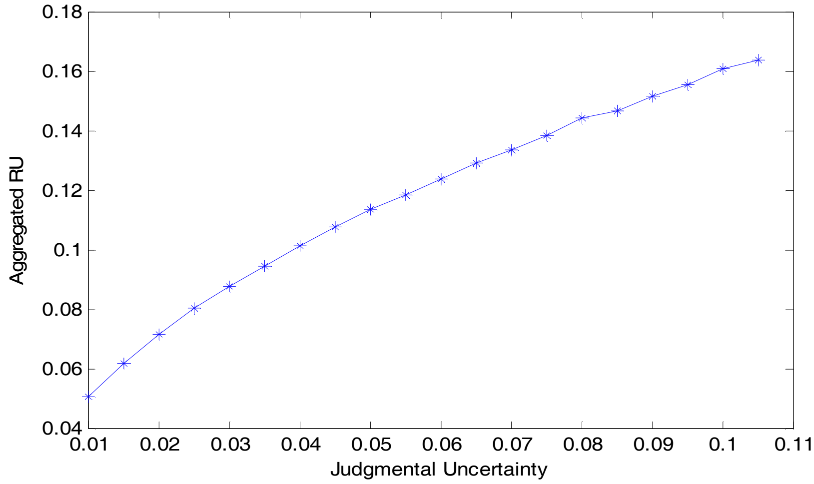
The simulation results, in Example 5.1, illustrates that, with only three runs, a required matrix, which is with the variance of its Eigen-vector does not exceed the limit for uncertainty, is obtained. Hence, there is no need to carry on the 100 replications. In case, if, for all 100 runs, there is no such possibility of obtaining a required matrix then the algorithm informs the replacement of the preference matrix by a new one and then re-start. This case was practised when the value of $\sigma = 0.01$; the simulation runs for all 100 replications did not satisfy the requirements, therefore, it was needed to change the judgmental matrix to re-start the implementation of the procedure for Algorithm 5.1. Thus, the implementations of this algorithm show that it is a flexible approximation technique to handle the problem of uncertainty.

Furthermore, with this algorithm, for the generated Eigen-vectors W_i, it is possible to calculate an aggregated measure namely; Rank Uncertainty (RU). This measure insures that, the judgmental uncertainty in each matrix contributes to the uncertainty in the final rankings of the decision alternatives; this measure is defined by Saaty (1980) and a formula to determine it is given by:

$$RU = \sqrt{\frac{1}{n}\sum_{1}^{n}(\sigma_i / w_i)^2} \tag{5.1}$$

If the same example, Example 5.1, is considered and assumed that the value for σ varies from $0.01 - 0.1$, with an increment value of 0.01; by adding another step to the same algorithm, then, from (5.1), the associated rank uncertainty, for each value of σ, for each W_i, can be determined. Hence, another run for this algorithm with the additional step was implemented and the simulation results indicated that when the value of σ is increased, it results in increasing the average uncertainty for the whole process; this result supports the same conclusion obtained by Wu (2007). The simulation run for Example 5.1, and different values of σ, to determine rank uncertainty, is illustrated by Figure 5.2.

Figure 5.2: Behaviour of RU as σ increases



Algorithm 5.1 was implemented considering various numbers of replications and different values of σ and applied to matrices with other dimensions, $4x4$ and $6x6$, the obtained results revealed exactly the same conclusions. Therefore, the technique described by this algorithm is a simple approximation one that is flexible to be incorporated with an AHP-based approach to reduce the weakness of the method.

Remark 5.3

To model uncertainty, there exist a considerable number of methods or theories (for example, probability theories) that make assumptions about available information based on a set of axioms. Research is generally done in the frameworks of these axioms and it is rare to find empirical techniques that can deal with uncertainty or to restrict it to an accepted level. Thus, the new algorithm in this chapter contributes in providing a novel tool that, empirically, is capable of controlling uncertainty to comply with the level of acceptance, which is specified by each user of this algorithm.

Therefore, if an AHP-based model, with a specified level of uncertainty is considered hence it would be combined with the proposed preference models to introduce a new preference-based strategy, for ranking preferences, that can handle quantitative in addition to qualitative criteria for scenarios that concern financial investments. The new AHP-based method is explained.

5.4 An AHP-Based Method with a Specified Level of Uncertainty

In this research, as explained before, methods for analyzing preferences, for solving decision problems, have been studied; they help decision makers make a choice between a pair of uncertain alternatives (lotteries), based on MVA, consistent with EUT, and cumulative distribution function using simulation. Two such models have been introduced. With the first one, a preference model ranks pairs of uncertain alternatives according to the determined values of the expected utilities, calculated from the computed values of the mean and variance for each of the generated alternative. The higher expected utility gives the higher ranking order. The second model ranks pairs of normalized lotteries with the same expected values based on determined variance for each of the risk factors defined as the ratio of the random variable (alternative) to its mean. Each risk factor is obtained from a multiplication decomposition of a random variable from one attribute structure in to two, the mean and a risk factor.

In both cases, ranking preferences are achieved objectively using the determined values of the expected utility, which are obtained from the implementation of the simulation algorithms; it is applied to deal with scenarios where only non-negative random variables are considered. However, the subjective consideration and judgments of the decision maker, which may have great influence on the decision process, have been ignored.

Therefore, in order to deal with the shortcomings of such approaches, in this research study the proposed ranking models, explained earlier, are modified; they are combined with AHP approach to conduct a new ranking strategy. It is based on AHP mainly because of its capability to handle both qualitative in addition to quantitative criteria in a way that can reduce the time and effort in making decision. Furthermore, the main procedure for this new technique can easily be understood and applied by decision makers; with the help of the software "Expert Choice" it can easily identify the priority of all criteria and sub criteria then determines the best option. In addition, the results can be transferred for easy computations that lead to arrive at a consensus decision.

However, the ranking procedure for the new AHP-based model is based on an integrated measure scale, which is resulted from the combination of the two modified approaches, the preference and AHP. In this combination, for each alternative, the determined value for the expected utility is converted in to a unit measure or weight that is incorporated with the one obtained from processing AHP to conduct an overall aggregated weight. This new overall scale, which is a single weight measure or priority for each option, is computed from the multiplication of the converted expected utility by the one that is obtained from applying AHP; it acts as a measure scale for ranking alternatives. Therefore, the new model described under this approach is an AHP-based model with distinction that:

1. For each comparison matrix, it limits the uncertainty to a desired level, using Algorithm 5.1, which is described before;
2. The overall weight, for each alternative, is determined from the multiplication of two measures from different sources, first one is resulted from the existing models, preference models, and the second measure is obtained from implementing the AHP.

Remark 5.4

The proposed approach in this chapter, which is based on the analytical framework of AHP combined with the preference ranking models, is an approximation framework that can be used as a structured procedure for ranking pairs of alternative options. It can construct the objectives and then synthesizes decision maker's measures, tangible and intangible, with respect to various criteria included in the decision process, and then evaluates the overall weight for each alternative option. The main contribution of this approximation approach is in controlling the limit of uncertainty within each preference matrix, which can weaken the decision maker's confidence in the results of the AHP. It provides, from the integration of two different approaches, a more accurate weight indicator that allows ranking preferences efficiently. The simulation-based model, which is proposed under this approach, can be applied to the field of financial economics. A case study, The Tender Selection Process (TSP) in Kurdistan Region of Iraq, is investigated testing the new proposed model. This will be explained in detail in the next chapter.

5.4.1 Overview of the model and the methodology

The main objective for this study is to establish a new preference ranking strategy, based on AHP, for ranking pairs of alternatives with non-negative outcomes only. The new model combines an existing model, a modified MVM, which was proposed earlier in this study, with an AHP-model, which is with a limited uncertainty, using simulation. In this section, to construct the objective structure of the model to produce a systematic procedure for ranking pairs of alternative option with non-negative outcomes only, an overview and the methodology for the new proposed model, an AHP-based model, using simulation, are explained. In this study, the main goal and levels for the hierarchy are introduced then the significant components, criteria and alternatives, are identified and the role played by each one, by taking in to account the determined values, which are obtained from implementing previous algorithms, is explained. For this model, pairs of simulated random variables are considered where ranking procedure will be according to the described methodology. Then, this model will be applied to a case study concerning an important process for financial investment, construction industry, which is the TSP.

Therefore, the start up step should identify the problem and the main goal. Here, the problem is ranking pairs of alternative options based on various, tangible and intangible, factors and the main goal is defined to be the selection of the best option, taking into account all quantitative criteria in addition to the subjective judgments of all experts who take part in the decision process. Hence, the next step, following this definition, involves the construction of the hierarchy structure; three main levels are identified. The first level is the main goal, the second is the level of criteria and sub-criteria; it is important here to identify the sub-criteria (if exist) and show their interrelation with the criteria, then illustrate their arrangements within the hierarchy. For the hierarchy in this study, it is assumed that a limited number of criteria are interrelated with a pair of alternatives on the third level. Then, preference matrices of pair-wise comparison are constructed; each represents a preference judgments of the alternative options, in the lower level, associated with a criterion identified in the level above; using relative scale measurements (1-9) to represent the decision maker's judgments.

Using simulation, for each comparison matrix, Algorithm 5.1, which was described earlier in this chapter, is applied to identify the matrix with a limited uncertainty, identified by the decision maker. The process could be started at the bottom level and move upward. Then, based on AHP axioms, ratio scales are derived and priorities are synthesized from the second level down by multiplying local priorities by the priority of their corresponding criterion in the level above, and adding them for each element in a level according to the criteria it affects. When the aggregated weight, for each option, is computed; it is multiplied by the first measure scale resulted from the computation of the expected utility, this gives, for each option, an overall weight scale; this will be the basis for ranking a pair of alternative options in the final stage.

The following algorithm can explain, in more detail, the proposed model and its procedure.

5.4.2 Algorithm 5.2

To clearly present the proposed model framework, this stepwise algorithm is described:

Step 1: Define the decision problem and the main goal;

Step 2: Form a team of decision makers (experts) collect all possible information about the decision problem;

Step 3: Structure the hierarchy from the top through the intermediate to the lowest level;

Step 4: Construct, for each level, except the final, starting from the level before the final one, the comparison matrices using steps, Step 4.1 to 4.4, below;

Step 4.1 Construct matrices for pair-wise comparisons, with one matrix with respect to each element, criterion, in the level immediately above it, using ratio scale measurements; the order of the matrix at each level depends on the number of elements at the lower level that it links to. Then the subjective judgments for the intensity of importance are given by decision makers, using a nine-point scale. If two objects are of equal importance then a value of 1 is given in the comparison matrix while a 9 refers to an absolute importance of one object over the other. The scale for entering judgments is given by *Table 2.2*, explained earlier.

Step 4.2: Generate, for each constructed matrix, a reciprocal normal random matrix to represent such matrix, using *Algorithm 1*. This new reciprocal matrix is with a specified level of uncertainty σ; the value of σ is identified, by the decision maker, at the beginning of the process;

Step 4.3: Estimate the relative weights, importance, for each decision factor using the Eigen-value method; the principle Eigen-vector correspond to the largest Eigen-value of each matrix constitutes the estimation of relative weights, local priorities. Expert Choice software can be used to calculate the Eigen-vectors and provide visual representation of overall ranking on a computer screen;

Step 4.4: Calculate the consistency for the preference matrix; a matrix is consistent if the numbers of the factors include in the decision process is approximately equal to the maximum Eigen-value; i.e., $\lambda_{\max} = n$, n is the number of criteria for each level. A consistency index CI, measures the inconsistencies of pair wise comparisons, is given by:

$CI = \dfrac{\lambda_{\max} - n}{n-1}$, where n is the number of criteria for each level.

Then, the consistency of the comparison matrix CR is:

$CR = \dfrac{CI}{RI}$, RI is the random index; its value depends on the order of the preference matrix, Table 2.3 shows different values for consistency index. The acceptable CR range varies according to the size of the matrix. The value of 10% or less is acceptable for CR.

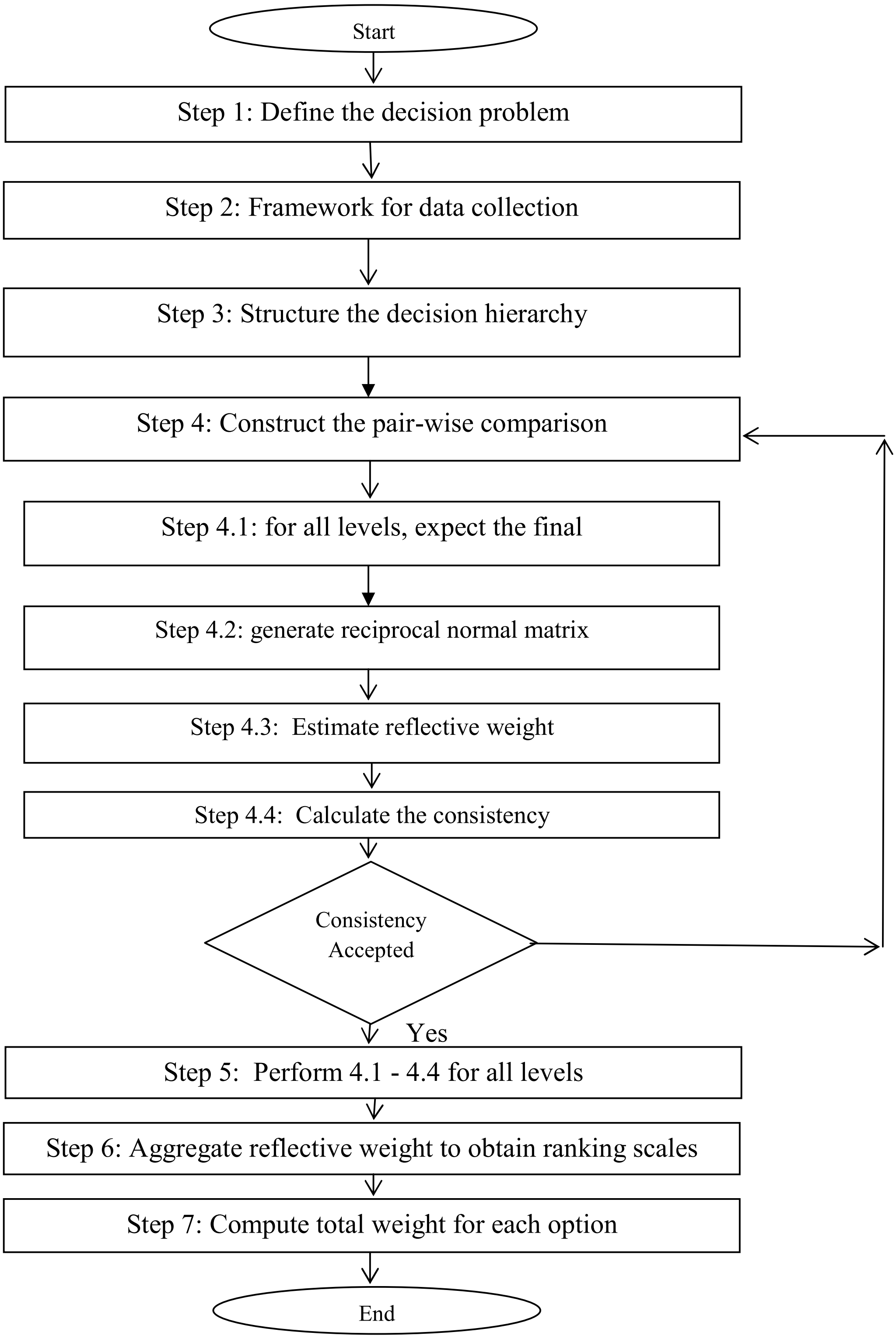
Step 5: Perform 4.1-4.4 for all levels in the hierarchy;

Step 6: Aggregate relative weights of various levels, obtained from Step 4.3, to obtain the ranking scale for the decision alternatives;

Step 7: Compute, for each alternative, the total weight, which is used for ranking, from the multiplication of the relative weight obtained from Step 6 by its corresponding one determined from the expected utility, computed earlier.

The main steps for this algorithm, Algorithm 5.2, can be illustrated by the following flow chart.

Flow chart 5.2: Algorithm 5.2 Flow Chart



Remark 5.5

In this chapter, the novel approach that is described by Algorithm 5.2, which is based on Algorithm 5.1, is considered as the core contribution. The described steps allow the decision maker build up a new approximation model that combine two models for different approaches; first is a modified MVM and the second is an AHP-based one. In this new combination, the integrated approach provides a simulation-based procedure for aggregating option weighting and forming a preference ranking strategy that can easily handle information, both qualitative and quantitative. Moreover, the use of the widely used software "Expert Choice" is another advantage of adhering to AHP axioms; this is a powerful software that can easily calculates the Eigen-vectors and provide visual representation of overall ranking on a computer screen. However, the main concept of this approximation approach is to extend the function of AHP, as for many other selection approaches that are based on AHP to provide a final weight scale to be based on in the selection process, for example, Yang and Lee, 1997; Escobar and Jimenze, 2000; Tam and Tummala (2001); Shapira and Goldenberg (2005); Shibatal et al (2009), but to introduce a more accurate weight indicator in terms of identifying the best option; it is based on two different approximation scales from different sources; if one of the indicators is not an efficient scale then the other one can compensate the deficiency. Therefore, this novel approach offers an efficient approximation tool that can guide decision maker in making logical decisions to solve large-scale decision problems especially those concern financial investments.

5.4.3 Applications of the model

At this stage, the proposed framework for the new approximation model in this chapter, rather than only being a theoretical concept must be a practical pursuit; it should be capable to handle or solve today's real-life problems. In addition, it is essential to consider the cooperation of individuals who are involved with such problems by using their own opinion and judgments to guide the model towards a satisfactory achievement. Therefore, another objective is to illustrate the applicability of this model, as a structured procedure, to a field of financial investments, construction industry. In the next chapter, the concentration will be on the construction project investments in Kurdistan Region of Iraq, and an attempt to conduct an empirical case study to investigate a specific case, which has significant influence on the success of implementing projects, the Tender Selection Process (TSP). Then the proposed model will be applied for selecting the most qualified tender in KR.

5.5 Summary

In this chapter, in order to deal with decision problems not only objectively, when judgments and experiences of experts are needed to be included in the decision process, especially when decisions include a great deal of losses and gains, a new approach was introduced; it proposed an AHP-based model for ranking pairs of alternatives option with non-negative outcomes. Meantime, in order to control the weakness of AHP results from the difficulty of setting up the right preference matrix to represent decision maker's judgments, the study described an algorithm. Using simulation, the algorithm restricted the uncertainty, which is considered as the average of the resulted variances associated with mapping the judgments to exact numbers, to a specific value, and then the rank uncertainty of the proposed algorithm was ensured. Then, to test the applicability of the designed algorithm to real life problems, an example was considered; the obtained results indicated the power and the flexibility of the algorithm. Hence, the new approach combined an existing preference ranking model, based on MVA consistent with EUT, explained earlier, with an AHP model, which was with a desired limit of uncertainty to introduce a new AHP-based model. Hence, the overview of the proposed model with the methodology was explained. A new algorithm, which describes the main assumptions of the proposed model, was introduced. The new approximation model, which included all influencing factors, provided an aggregated weight scale, obtained from multiplying two weight scale; the first value was the expected utility determined from applying the preference ranking model, and the second was resulted from implementing the AHP-based model. The integrated measurement obtained under this approach, which is obtained from different sources, yielded more accurate results in terms of identifying the best option; it had the capacity to handle a great number of criteria and allowed incorporating judgmental perceptions for experienced professionals. In addition, it offered an efficient approximation tool that guided decision maker in making logical decisions to solve large-scale decision problems especially those concern financial investments-scenarios.

CHAPTER SIX: THE INTEGRATED AHP-BASED MODEL AND APPLICATIONS

In decision making problems, to evaluate and rank pairs of alternative options with respect to an objective, preference ranking procedures have been suggested. Each of such modelling procedure, rather than only being theoretical concept must be practical act; its flexibility and usability should be demonstrated in a field of applications to solve real-life problems with the people who have these problems. In order to ensure the applicability of the new proposed approach, which is explained in the previous chapter (Chapter 5), as a structured procedure to solve a real-life problem, this chapter links the theoretically developed model with the deductively constructed criteria, which is constructed for the same purpose. It applies the framework to a field of applications, which is construction industry, a specific case that has significant impact on the success of projects implementation; the Tender Selection Process (TSP) in Kurdistan Region (KR) of Iraq. To demonstrate this, despite that research in the area on TSP is limited, this chapter explores the first empirical case study, which describes the first comprehensive and inductive study, on TSP that has received a minimum consideration in this region. It investigates, through conducting a pilot study, various construction companies, their type, size, classification and other information. The constructed questionnaire, based on interviews with a representative sample of construction experts, content validates and completes by a sample of construction experts in KR. Hence, based on the obtained information, it conducts the main survey; the final questionnaire includes detailed information on the construction organizations and their experiences on tenders. Finally, based on results of the survey, it identifies the main criteria, which are believed to have significant impact on TSP in KR, with the evaluation of their weights and then it verifies the reasons that may cause the delivery problems for the implementation of the entire projects. A real-world example, which is TSP in KR, demonstrates the applicability of the proposed framework.

6.1 Introduction

Tender Selection Process (TSP), which is the process of selecting the optimum tender/client among the list of all alternatives to deliver a successful construction project, is a significant decision. (Clients refer to the construction companies or those persons investing in the construction sectors). In the selection process, for identifying the most qualified tender, in addition to the tender price, various criteria such as qualification and skills of the contractor, past experience and performance, financial capability and the subjective judgments of the experienced experts should be considered simultaneously. Moreover, in order to evaluate the capability of the contractor based on specific requirements of the project under consideration, it is necessary to propose selection methods that provide systematic methodology to incorporate the bid price simultaneously with these decision criteria.

However, in each country, the selection of an appropriate procedure for evaluating tenders requires an extensive and comprehensive analysis of the project characteristics, the main criteria to be considered, and specific requirements of the client, the goal and objectives. Based on these characteristics, a list of criteria can be generated.

Therefore, the main reasons for inadequate tender selection process can be described as an inappropriate selection of:

- The criteria for evaluating the qualification of the tenders;
- The importance attributed to these criteria;
- The methodology applied for tender selection process and its evaluation.

Hence, in order to select a qualified contractor to achieve a successful construction project that meets all requirements; the qualification must be evaluated by defining an appropriate method for the evaluation of criteria. Multi-criteria evaluation methods may be used in evaluation of contractor bids and weighting criteria can be determined based on the priorities of the client. The review of literature revealed the existence of various evaluation methods, one of the most popular methods is the Analytic Hierarchy Process (AHP) introduced by Saaty (1980).

In this chapter, in order to follow an inductive approach to construct best tender selection strategies in Kurdistan Region (KR) of Iraq; first, it is needed to investigate construction organizations/clients then explore TSP in the region.

6.1.1 Construction organizations in KR

Construction organizations/Clients, in KR, represent all organizations that perform construction projects, housing; roads and bridges; railways; airports; sea ports; irrigation projects in addition to extending and renovation projects; they are organized within two main sectors:

1. Public (Governmental Organizations);
2. Private (Private Construction Companies).

The public sector comprises all construction departments/ sections within all ministries. For this sector the selection of qualified construction client, by means of bidding mechanisms is regulated by special laws issued by authorized institutions from the regional government.

The private organizations, including all private construction companies, are classified, according to a combination of their qualifications; financial capability and assets; past performance; past experiences, into 1-10 grade companies from the highest to the lowest. For this sector the selection process is not needed to meet the law requirements while these laws are only used as guidelines; clients can devise their own evaluation criteria and form their own priority list.

Moreover, for both sectors, there is still no structured procedure that can help in evaluating the skills, qualifications and capabilities of the contractor, in comparison with the specific criteria and requirements necessary for each project under consideration.
However, in KR, which is experiencing a construction boom incomparable to any other parts of Iraq, during the last decade, a stable security situation backed by new investment laws that are known to be investor friendly, has created an attractive business environment for economic growth. As part of construction development plan for the Regional Government, recently, the region has witnessed massive construction projects and a growing level of developments; investors from all over the world have become involved in major projects. Despite the massive project achievements in the region, there is no minimum standard that can guaranty the quality of the TSP with no structured formalized procedure to be practiced to evaluate tenders. Furthermore, in this region, due to the lack of the research studies that have been carried out on this process, there exist no published data that can be used as primary information for conducting structured methods to evaluate tenders; it depends, to a great extent, on the knowledge and skill of the clients.

Therefore, in order to construct best tender selection strategy in KR, it is first needed to identify the key issues. Thus, the aim of this chapter is to investigate TSP and tendering procedures and then identify the main criteria that are believed to have significant impact on the success of the selection of the most qualified tender in addition to the main reasons that cause the failure of the implementation of the projects in this region. This will enable to bridge the theoretically developed models in the previous chapter with the deductively constructed criteria for tender selection.

Hence, to identify the main factors concerning TSP in KR, a pilot study is conducted; a questionnaire is designed and then content validated based on interviews with a representative sample of construction experts covering the three governorates for public and private sectors. The final survey, based on the obtained information, is conducted; the final questionnaire includes detailed information on the construction organizations and their tendering procedures. Results of the survey are used to identify the main criteria for the evaluation of tenders, their qualifications, importance/weights and the main causes of delivery problems. In this chapter, following review of the literature, the main issues are explained.

6.2 Literature Review

Construction projects often start with clients making project proposals as a result of the announcement of a tender. In this process, clients make numerous decisions which may result in the success or failure of the entire project. One of the most challenging decisions is the task of selecting an appropriate tender or contractor (bidder) for the project. In this, various research studies, concerning TSP, have been introduced; they focused on the main elements such as different procedures that have been implemented to identify a qualified tender, for example, Friis (1987); Russel et al. (1990); Palaneeswaran and Kumaraswamy (2001), or evaluation methods that has been used for contractor selection, for example, Fong and Choi (2000); Cheng and Li (2004); Halil (2007), others to identify common criteria to evaluate contractors, such as, Holt et al. (1994); Hatush and Skitmore (1998); Banaitiene and Banaitis (2006), or reasons that cause the failure of the projects, Lim and Mohamed (1999); Laryea and Hughes (2008); Al-kharashi and Skitmore (2009).

Furthermore, this review revealed that this process accepts the principal of the lowest bid, which is mostly described as the key function for evaluating and winning a tender, Wong et al. (2000); Ling (2005). However, there is a realization that the lowest price does not necessarily achieve the best selection and lowest bidders have failed to complete projects; Banaitiene and Banaitis (2006) provided sample attitudes cited by researchers since 1967 concerning the influence of the tender price on the final selection. Most of them agree that a qualified contractor should be selected based on a value for money basis, which would enable the client to evaluate tender's capability not only according to the lowest price but also according to other quantitative and qualitative criteria (Samuelson and Levitt, 1982; Russel et al., 1990; Crowley and Hancher, 1995; Herbsman, 1995; Wong et al., 2000; Mahdi et al., 2002; Halil, 2007; Laryea and Hughes, 2008). A further delivery problem is that there are no minimum standards that guarantee the quality of the selection; it depends, to a great extent, on the knowledge and skills of the expert whose decisions varies from one to another (Alhazmi and McCaffer, 2000; Mahdi et al., 2002; Al-kharashy and Skitmore, 2009).

In this chapter, in order to apply the theoretically developed model, which was explained in the previous chapter, as a structured procedure for TSP in KR and link it with the specific criteria that are constructed for the same purpose, a pilot study is conducted. The aim is to identify, through an investigation, construction organizations in the region and the tendering procedures that have been practiced then verify the specific criteria that are believed to be significant for the selection of the most qualified tender in the region. Furthermore, this study investigates critical reasons/criteria that cause delivery problems, which may lead to the failure of the entire project, and then it determines the importance/weights of these criteria. The main results are used as the primary data for conducting the main survey. The following sections explain the aims and objectives for the study and then describe the data analysis and the main results and conclusions for the pilot study before conducting the main survey.

6.3 Aims and Objectives

The aim of the study in this chapter is to investigate TSP to select a qualified tender to achieve a successful construction project that meets all requirements, using the developed AHP-based model; the qualification should be based on specific criteria with their estimated weights or importance. Therefore, the main objectives for this investigation study are to:

1. Identify the most significant criteria, to be based on, in selecting an appropriate tender;
2. Assign weight to each criterion;
3. Verify reasons that cause the failure of the construction projects.

In order to achieve these objectives, the following tasks need to be considered:

1. A comprehensive review of literature about the key issues for TSP;

2. Collection of data to conduct a pilot study using a structured questionnaire;
3. Analyzing the questionnaire to obtain the main result;
4. Collection of data, based on the obtained results, to conduct the main survey;
5. Arriving at a set of conclusions and recommendations.

Therefore, in order to achieve the main objectives of this chapter, a pilot study, as an early study to prepare for a major one, is conducted to investigate the process of tender selection for construction companies and evaluation of criteria. Based on a comprehensive literature search, to identify the key issues or to determine the main criteria necessary for a best tender selection process in KR, a structured questionnaire is designed to conduct initial data for the primary outcome measures. Before organizing the proposed questionnaire to conduct the initial data for this survey, a pilot study and the main reasons for conducting it are explained.

6.4 A pilot study

A pilot study, or a feasibility study, is a small study in comparison with the main one; it is an early preparatory study to test logistics and feasibility for collected data; it may provide significant information prior to a larger study. In order to design a project, a pilot study can reveal problems and deficiencies that may lead to significant changes in the design of the study; these can then be addressed before time and resources are expended on large scale studies. Therefore, a clear list of aims and objectives for a pilot study is very important.

However, the main logistic issues, which can be addressed by a pilot study prior to the main study, are to check the:

- Comprehension of the main categories and instructions given in the questionnaire;
- Skill and qualification of the investigators;
- Reliability and the validity of the questions and their results.

6.4.1 Investigation objectives for this pilot study

In this study, the investigation has the following main objectives:

- Investigate the type and the classifications of the construction organizations;
- Investigate the tender selection procedures;
- Conduct the main evaluation criteria suggested by the investigators;
- Evaluate the importance of the evaluation criteria;
- Identify most significant criteria for selecting the best tender;
- Identify crucial reasons for the failure of the implementation of the agreed tender.

To conduct the pilot study, a questionnaire is constructed and distributed to various construction organizations (both public and private sectors), with knowledge about tendering procedures, to identify the main factors concerning TSP. This questionnaire consisted of five main questions accompanied by a covering letter, was designed based on a combination of an extensive literature review in the area of the study. The details of a sample study, data analysis and results for such pilot study are given below.

6.4.2 Sample selection

In this study, a sample of 25 was drawn from the construction companies and governmental organizations (construction departments), within KR. The respondents/clients targeted were construction experts/contractors and experienced engineers within governmental ministries and construction company owners registered with planning ministry; they were selected according to their background and experiences in the construction developments.

A questionnaire has been constructed based on interviews with a representative sample of construction experts; experts refer to skilled engineers or contractors who have knowledge and experiences about construction projects and tendering procedures. The formulated questionnaire identifies 5 main categories. First one is about the identification of the construction organizations; it is optional as identities will not be released to any third party without their full consent and all individual responses will be kept confidential. Second category concerns the general information about each construction organization which includes type, classification of the organization, whether it is data protected or it is a subsidiary company for another larger company. Third category concerns the utilized tendering procedures, attitude towards evaluation, and evaluation criteria. While the fourth category investigates a list of common criteria, eight criteria, that are believed to be standard criteria for the process of tender evaluation and tender selection for many countries; these are set forth as the result of many studies, for example, Cheng and Li (2004); Banaitiene and Banaitis (2006). Respondents to this questionnaire are needed to rank these criteria according to their importance in KR, giving scale 1 to the most important criteria, 2 to the second most important, etc, and 8 to the least one. Finally, in category five, respondents are required to identify, based on their knowledge and experiences in the region, the main criteria, which are believed to be significant, to be based on in selecting the best tender and then specify the most significant reasons lead to the failure of the implementation of the agreed tender.

Hence, 25 questionnaires were distributed where only 15 (60%) completed questionnaires were received, of which 74% respondents represented public organizations and 26% represented private sector. The 60% response is considered satisfactory for a pilot study of this type; respondents who declined to participate, in the study, stated reasons such as luck of resources and experiences in conducting surveys or luck of interest. Similar reasons might be the cause of non-response to the main survey.

However, this preparatory study, which was completed by a sample of 15 of construction experts and contractors covering the three governorates of KR for public and private sectors, was very informative in conducting primary data for the main survey.

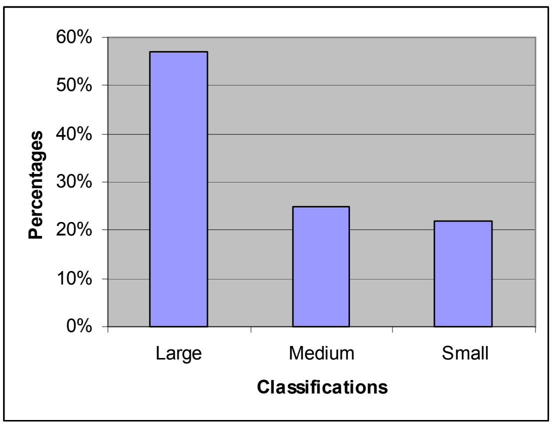
Thus, this pilot study, which is focused on the process of tender selection and evaluation criteria in KR, is a preparatory study to conduct and collect data then to test the feasibility of the data; it may provide information prior to the main study. The data analysis for the achieved pilot study is described.

6.4.3 Data analysis for the pilot study

In this study, 15 organizations of which 11 (74%) are public and only 4 (26%) are private, are classified into two types of clients. First is the public which represents clients from various construction institutions from different ministries within Kurdistan Regional Government while second type is the private sector that represents all non-governmental clients. Each type of the client, public or private, is classified into small, medium and large organization where Data Protection Act (DPA) has not been yet legislated; although there is a common understanding that the main components of this act are followed. In KR where most of these organizations are not subsidiary of another one, the system of classification is regulated by law which is based on different criteria such as the size of its capital and assets, past experience, past performance, registration within authorized parts and others.

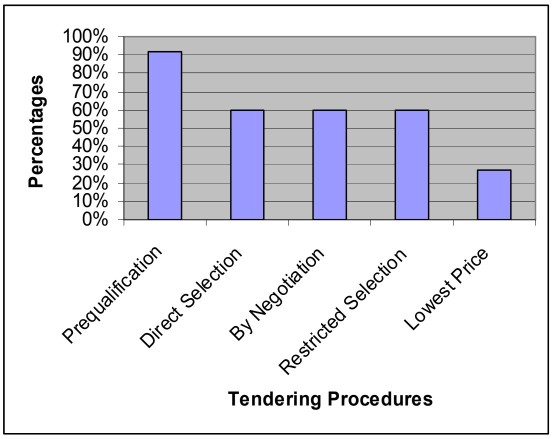
For this sample, for both sectors, the public and private, 58% are classified as large and 24% is classified as medium while only 18% are classified as small. Thus, most of the sample is comprised of large organizations; it is illustrated by Figure 6.1.

Figure 6.1: Classification of Construction Organizations



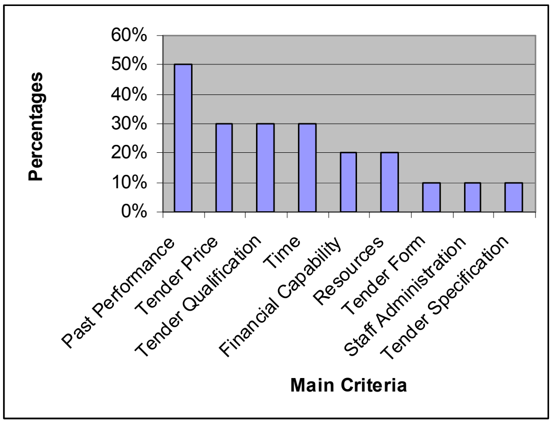
Hence, after describing the type and classification of the construction organizations, the respondents were asked what tendering procedure they use for their selection process. For this study, 92% of the respondents use pre qualifications; it is the process of determining the candidate's ability by comparing the key criteria, to meet the specific requirements, set forth by each company with only 27% based on the lowest price. 60% of the respondents use each of direct selection, by negotiation and restricted selection procedure. Thus, most of the respondents to this questionnaire use not only one procedure; multi-procedures are used according to specific requirements for each project. Figure 6.2 shows the responses to this procedure.

Figure 6.2: Tendering Procedures Practiced in KR



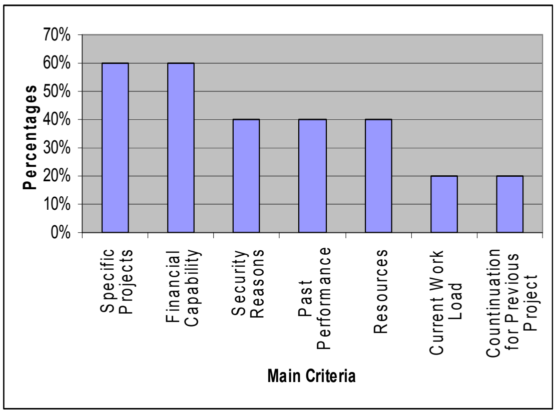
For the selection process, the respondents were asked to identify the main criteria to be based on, according to their importance, in using pre qualification. From 91% respondent to this procedure, 50% use past performance as the main criteria to be based on in the selection process and 30% use each of criteria, tender price, tender classification and time while 20% consider financial capability, resources as the main criteria while only 10% of the respondents think that each of organization of the tender form, staff administration and tender specification are the main criteria. It is illustrated in Figure 6.3.

Figure 6.3: Main Criteria for Prequalification Procedure



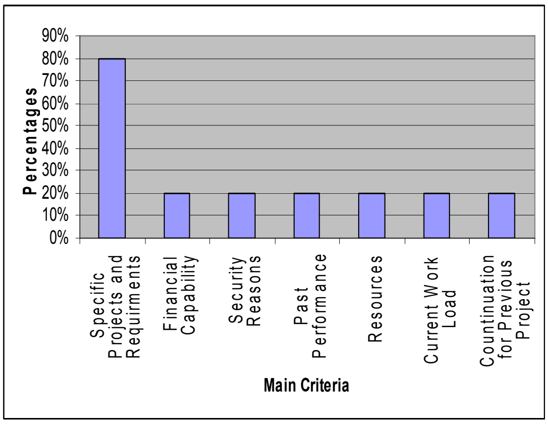
For direct selection, where only 45% of the respondents use this procedure, 60% think that each of specific projects, financial capability of the tender are the main criteria, 40% think that each of security reasons, past performance and resources are the main criteria while only 20% think that current work load and continuation for previously achieved project are the main criteria. It is illustrated by Figure 6.4.

Figure 6.4: Main Criteria for Direct Selection Procedure



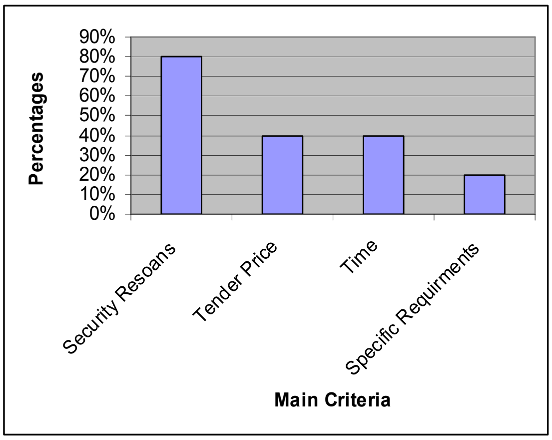
For Negotiation procedure, 80% of the respondents to this procedure think that specific projects and requirements is the main criteria while 20% think that each of financial capability, past performance, continuation for previous achieved project and current work load are the main criteria. It is shown by Figure 6.5.

Figure 6.5: Main Criteria for By Negotiation Procedure



For Restricted Selection procedure, 80% think that the main criteria is security reasons for very specific projects, such as construction of air ports; military buildings; and others, 40% think of tender price and time as the main criteria and 20% think that special requirements for some specific projects is the main criteria; it is illustrated by Figure 6.

Figure 6.6: Main Criteria for Restricted Selection Procedure



Finally, the process of evaluation of a contractor, based on various significant criteria, to select the most qualified one is considered. In this region, however, no empirical study has been carried out on this process and published data are not available to be used as primary data and then to build on new studies, this investigation relies on standard criteria that are considered as significant for other countries. It considers eight of the most important criteria according to their importance; these are resulted from conducting many studies for other countries, for a successful TSP (Cheng and Li, 2004; Banaitiene and Banaitis, 2006). The criteria listed are tender price; financial capability; past performance (this includes failure to complete contract, delay, cost overruns and actual quality achieved); past experience (this includes scale of project completed, type of project completed and experience in local area); resources (includes physical or technical resources and human resources); current work load; past relationship and safety management. In order to investigate the importance of each of these criteria, if it has been considered as significant in the selection process in KR as for the other countries, the respondents are required to rate them according to their importance.

The results for this study pointed out that 70% of the respondents agreed that the main criterion that has been practiced in the selection process is the tender price; it is based on the principle of the lowest price. Then, 40% of the respondents thought that the second most influence criteria are the past performance and past experience (both are considered as one criterion according to their importance) and resources for the contractor while the other criteria such as financial capability, current work load and past relationship has less influence. However, according to respondent's thoughts, safety management is not taken seriously as one of the main criteria.

6.4.4 Main conclusion

In this primary study, the main conclusion is the identification of the criteria that should be considered as significant, according to their importance, for a successful selection process in KR, and then determination of the percentages of agreements on each criteria; these are derived from the respondent's viewpoints, and it is explained by the following table.

Table 6.1: The most significant criteria and their percentages of agreement

	The most significant criteria	percentages
1	Tender's skills and qualifications	54%
2	Past performance and past experience	50%
3	Tender price	40%
4	Financial capability for the contractor	33%
5	Resources	27%
6	Current work load	14%
7	Qualified administration staff	7%

Moreover, despite that 70% of the respondents agreed that lowest tender price has been practiced as the main criterion for the selection process; only 40% agree that it is one of the main criteria and with ranking order as third. While the majority of the respondents believed that lowest tender price is the main reasons that led to the failure of many projects. Other reasons may be when the project is not implemented by the first client; it is transferred to the second or may be third party. Table 6.2 introduces other factors that were believed to be reasons for the failure of the project with their percentages.

Table 6.2: Other factors that cause the failure and their percentages

	Other factors cause the failure	percentages
1	Fluctuation of material prices	60%
2	Unqualified tender and his staff	53%
3	Political situation	34%
4	Security reasons	27%
5	Tender's financial problems	20%
6	No quality control for materials	14%
7	Lack of resources for the tender	7%

Remark 6.1

For this investigation, as there exist no empirical studies on this process, data from other studies for other countries are used for identifying the most significant criteria in KR, where the respondent were asked to verify each of such criterion if it is an important one, and then, to rank them according to their importance in the region. The main finding, from the data analysis for the conducted study, is the identification of the analogous criteria for the best selection process in KR as for other countries, for example, UK; USA; Australia; Canada; Saudi Arabia; Singapore; Lithuania; Malaysia, but with different ranking orders. Furthermore, in this region, due to the distinct circumstances, in addition to the practice of the lowest price, the main reason that leads to the failure of the entire project is incomparable to those for these countries; due to high volatility, the currency fluctuation is believed to be the main reason that fluctuate material prices; this has significant impact on the implementation of the entire project in KR.

6.4.5 Modifications and adjustments performed as the result of the pilot study

In research studies, the generations of items and data collection are the most important elements of establishing measures. Questionnaires are the most commonly used method, for data collection, designed to transfers the aims of the research into research questions which would be answered by knowledgeable respondents; these answers can be summarized in descriptive tables and the results could be extremely valuable for planners and administrators. However, these questions should be comprehensive, clear and concise, avoid any ambiguity, and identify the type of analysis that will be generated. Generally, the questionnaires that are clear, precise and easy to deal with tend to get higher response rate. Otherwise, analysis derive from the questionnaire will be misleading.

Therefore, to design a comprehensive and precise questionnaire it is necessary not to be expensive and time consuming; it is important to ask experts, who are knowledgeable about the subject matter, to make sure that most of the important questions have been addressed and that measure validation demonstrates the adequacy of these questions; in addition, it should normally be tested before its final administration. In order that measurements for a questionnaire may be assessed in a meaningful way, they should be valid and reliable. It is believed that assessment of reliability and construct validity of measurements are virtually meaningless unless it has ensured, to some degree of confidence, the content validity of measurement criteria and that no degree of reliability and construct validity can compensate for lack of content validity. Therefore, it is assumed that content validity will be assessed immediately after items have developed, this will provide the opportunity to refine or replace items before preparing and administrating a final questionnaire (Schriesheim et al, 1993; Houston, 2004; Jandaghil and Shaterian, 2008).

Furthermore, it is necessary to ensure not only the importance of content validity but also its formal quantitative assessment which corresponds to an evaluation of the accuracy and adequacy of measurement scales as a part of an empirical research process (Lawshe, 1975).

Therefore, in this pilot study, before the main survey was practiced, content validity of the designed questionnaire and its formal quantitative assessment, which are significant issues to ensure about the question "are we measuring what we think we are measuring", were achieved. The validation of the measurements in the questionnaire was determined to make sure that many of the important questions have been addressed and the questions that are not clear or potentially ambiguous have been identified.

6.4.6 Content validity

Validity refers to the accuracy or truthfulness of measurements for a questionnaire or whether it measures questionnaire (a survey) measures what it is intended to be measured and if the questionnaire is comprehensive enough to collect all information needed to address the purpose and goals of the study appropriate for the sample/population (Carmines and Zeller, 1979; Lancaster et al, 2004; Straub et al, 2004). However, for empirical research studies, it is important to validate the measurements in a questionnaire to see if these questions truly cover the content of all dimensions. The validity measurements can be evaluated from the evaluation of content validly that deals with the assessment of the validity of the questions; knowledgeable experts (academics practitioners) need to review the questions to ensure that they are representative, comprehensive and understandable or address the problems of interest. It is also need to conduct a literature review to ensure that the major items are considered in constructing questions in the questionnaire.

To investigate content validity of a questionnaire two approaches are available. First, it is judgmental or qualitative based on a high degree of consensus among expert judges; it requires researchers to be present with experts in order to assess validation. For this case study in particular, this approach is not practical to be employed as it is not possible to have many experts to participate; this makes validation to be limited by few experts. The Second approach is an empirical (statistical) or quantitative one that involves a statistical estimation for validity ratio. First quantitative approach, which estimates the statistical validity ratio, was introduced by Lawshe (1975); this approach determines the extent of overlap between a job performance domain and a specific test by using a content evaluation panel composed of experts knowledgeable about the process.

Therefore, for this case study, in order to evaluate the validity measurements, a small sample (sample of 12) of potential experts were asked to review the content of the questionnaire then to interpret their understanding of the questions in the questionnaire to comment on each item's expression and readability. Then, to investigate whether the identified constructs and their related issues sufficiently cover relevant dimensions of the criteria, which have great impact on the process. Hence, refinements were made based on their suggestions to obtain feedback for improvements to provide reliable measures to prepare for conducting the main survey.

However, for any empirical research, content validity is practiced; its application for the process of tender selection is limited, specifically, it is not yet employed in any studies related to this process in KR; therefore, it was the focus of the study in this chapter.

Therefore, in this study, for the validation process, the quantitative approach proposed by (Lawshe, 1975; Lewis et al, 1995) was utilized. The assessment of validation allowed the participants (respondents to the questionnaire) to investigate the extent to which these questions represent the content of the criteria being measured, then investigated the comprehensiveness, or the extent to which these questions cover contents of criteria and their related items, and the clarity of the questions, whether there is ambiguity in identifying these items; then the participant were asked to give suggestions on how to revise and re-write these questions to make them clearer prior to the main study. The respondents, who participated in this process, were asked to assess the significance of the criteria that have been adopted, as the main criteria, by several countries (UK; USA; Australia; Canada; Saudi Arabia; Singapore; Lithuania) and suggested by other researchers (Samuelson and Levitt, 1982; Fong and Choi, 2000; Banaitiene and Banaitis, 2006**)** to the process of tender selection in KR, in addition to identify the best suitable scale for weighting these measures.

Finally, each respondent was asked to give his judgment whether the five most significant criteria for selecting the best tender and the five most significant reasons for the failure of implementation of the agreed tender, obtained from the consensus of 20% top clients/contractors, are reliable and valid. From the consensus of participants, it was ensured that all categories, which were addressed by the questionnaire, are essential to be included in the final form; with regard to the suggestions concerning the refinements and rewording of the items; these suggestions incorporated when the new questionnaire for the main survey designed.

6.4.6.1 Conclusion and suggestions

In this study, it is concluded that, respondents to the validation process supported the content of survey instruments of the designed questionnaire utilizing the quantitative approach (Lawshe, 1975; Lewis et al, 1995); they suggested rewording few items for the questionnaire preparing to conduct a more general one for the main study that will investigate the process of tender selection in the main three governors in KR. Hence, the data, which was obtained from the conducted pilot study, was very informative and then considered as the primary information for designing a questionnaire for the main survey, which is explained in the following section.

Remark 6.2

Research studies concerning TSP, in KR specially and Iraq in general, have received a minimum amount of consideration; it is not yet employed in any studies related to this process; this generates the lack of interests for professionals and experienced experts in conducting surveys to formalize knowledge. Therefore, the main contribution of this study is the identification of the main key issues that concern TSP; it is considered as the first informative study for the formalization of knowledge in KR.

6.5 Survey

In KR of Iraq, due to lack of research studies and published data in addition to the deficiency in existing methods for evaluation of criteria, both sectors, the public and the private, have not yet developed their own well structured formalized procedure to evaluate contractors; they may have subjected themselves to the risk of selecting contractors with inadequate skill, capacity and experience into the bidding process. In order to formulate a structured policy, which is highly critical, to improve the system of evaluation of bids, for a successful process of tender selection in the region, a main survey is conducted. It is considered as the first informative study to be achieved for conducting primary data; it is focused on TSP and evaluation of criteria. For this purpose, a questionnaire survey, based on data extracted from the first study, is designed then distributed to various construction organizations, which have knowledge about the tendering procedure, to identify, according to their opinion, the most significant criteria that have great impact on the selection process, then to verify the main reasons that might lead to the failure of the process. The details of a sample study, data analysis and results for such survey study are given below.

6.5.1 Sample selection

A study sample of 60 was drawn from the construction organizations within KR. The choice of the sample was made on the basis of representativeness and the clients targeted were construction experts within ministerial council and universities, experienced engineers within governmental ministries (public clients) and construction company owners (private clients, local and international); they were selected according to their background and experiences in the construction developments. During this study, all the organizations were contacted and the aims and objects for this questionnaire survey were explained, then the representative for each client was asked to fill in a questionnaire form. The survey questionnaire form consists of five main categories, namely:

1. Identification of construction organizations, this includes name of organizations, name of authorized representative, contact position (job title), address (authorized representative), telephone and fax number, email address (authorized representative), web site (if any), organization registration number and date of registration, this category, in which identities will not be released to any third party without their full consent and all individual responses will be kept confidential, is optional;
2. General information on the organizations, this represents type of organization (public, private or other), classification (small (S), medium (M) or large (L)), local or international, registered under Data Protection or not, subsidiary of another one or not;
3. Contractor evaluation and contractor selection, this includes what tendering procedures are employed and what are the most significant criteria, according to their importance, to be based on for each tendering procedure, with two answers for each one; Yes or NO. Then if the answer is No, from a list of main criteria identified from analyzing the pilot study (previous questionnaire) to be based on for the selection process, specify the degree of agreement: strongly disagree; disagree; neither agree nor disagree; agree; and strongly agree. These procedures are: (1) pre-qualifications? If Yes, is the principle of acceptance based on the lowest price? Again the answer is

Yes or No, if No, from a given list of most important criteria (tender price, time, tender classification, past experience, past performance, financial capability, well – organized tender form and finally other criteria) indicate the degree of agreement. (2) Direct selection (Direct invitation) with two answers, Yes or No, if Yes, are the projects very specific? Then the answer is either Yes or NO. If No, from a given list of most important criteria (security reasons, resources, financial capability, past relationship, past performance continuation for previous projects, tender price and finally other criteria), what is the degree of agreement. (3) By negotiation with two answers, Yes or NO. If Yes are the projects very specific? Again the answer is Yes or No, if No, from a given list of most important criteria (security reasons, special requirements, tender price, resources, financial capability, past relationship, past performance, continuation for previous projects and other), then what is the degree of agreement. (4) Restricted selection with two answers, Yes or NO. If Yes are the projects very specific? and the answer is Yes or No, if No, from a given list of most important criteria (security reasons, financial capability, past relationship, past performance, continuation for previous projects and other), then indicate the degree of agreement from five given degrees;

4. Identification of the most significant criteria for selecting the best tender. In this category, there is a list of criteria, according to their importance, identified by the expert’s information provided by the pilot study, to be based on in selecting the best qualified tender for a successful tender selection process; these criteria are lowest tender price; tender’s skill and qualification; financial capability for the contractor; past performance and past experience; resources; current work load; currency stability; material price stability; governmental support and others. It is required to indicate a degree of agreement (1-5) to the importance of these criteria;
5. Delivery problems. This investigates the most significant reasons, which cause the failure of the implementation of the agreed tender. A list of criteria is provided which includes lowest tender price; tender's financial problems; unqualified tender and his staff; political situation; unsettled prices; lack of governmental support; tender's work load; transferring tenders to other parties; no quality control for materials; and other.

For each one, then it is required to indicate a degree of agreements (1-5)

Hence, 60 questionnaires were distributed to various construction organizations for both public and private sectors where only 43 (72%) completed questionnaire forms were received. The participants to this survey were owners of private construction companies and consultants or experts in public-sector organizations, in addition to experts from different universities in the three main governors in KR, but with two different financial regulations. Therefore, the three governors in KR are considered as two main regions or cities, first is “Erbil and Duhok” and the second is “Sulaimani” region. The data analysis and the results for this study are explained.

6.5.2 Data analysis and results

In this investigation, the collected data, which was obtained from the main survey, is analyzed using SPSS Version 16 and the conclusions are based on these analyses. The following tables, Table 6.1-6.7, give an overall results for the analysis of data, according to the city; classifications for the type of organization and tendering procedures for all participants from both regions for both sectors, public and private.

Table 6.3: Type of Organizations for Regions; Class Organization

Type of organizations				Class of Organization S	M	L	Total
PUBLIC	**Region**	**ERBIL & DUHOK**	**Count**	**5**	**2**	**8**	**15**
			% within Region	33.3%	13.3%	53.3%	100.0%
			% within Class Organization	83.3%	28.6%	66.7%	60.0%
			% of Total	20.0%	8.0%	32.0%	60.0%
		SULAIMANI	**Count**	**1**	**5**	**4**	**10**
			% within Region	10.0%	50.0%	40.0%	100.0%
			% within Class Organization	16.7%	71.4%	33.3%	40.0%
			% of Total	4.0%	20.0%	16.0%	40.0%
	Total		**Count**	**6**	**7**	**12**	**25**
			% within Region	24.0%	28.0%	48.0%	100.0%
			% within Class Organization	100.0%	100.0%	100.0%	100.0%
			% of Total	24.0%	28.0%	48.0%	100.0%
PRIVATE		**ERBIL & DUHOK**	**Count**	**1**	**4**	**7**	**12**
			% within Region	8.3%	33.3%	58.3%	100.0%
			% within Class Organization	100.0%	80.0%	58.3%	66.7%
			% of Total	5.6%	22.2%	38.9%	66.7%
		SULAIMANI	**Count**	**0**	**1**	**5**	**6**
			% within Region	.0%	16.7%	83.3%	100.0%
			% within Class Organization	.0%	20.0%	41.7%	33.3%
			% of Total	.0%	5.6%	27.8%	33.3%
	Total		**Count**	**1**	**5**	**12**	**18**
			% within Region	5.6%	27.8%	66.7%	100.0%
			% within Class Organization	100.0%	100.0%	100.0%	100.0%
			% of Total	5.6%	27.8%	66.7%	100.0%

Table 6.1 shows that, from the total number of participants, 25 (58%) are from public organizations; 15 (60%) of them are from Erbil and Duhok, in which 5 (33.3%) are small (S), 2 (13.3%) are medium (M) and 8 (53.3%) are large (L), and 10 (40%) are from Sulaimani, with 1 (10%) is S, 5 (50%) are M and 4 (40%) are L. Private companies comprises 18 (42%); 12 (66.7%) of them are from Erbil and Duhok, of which 1 (8.3%) is S,

4 (33.3%) are M and 7 (58.3%) are L, and 6 (33.3%) are from Sulaimani of which only 1 (16.7%) are M and 5 (83.3%) are L.

Table 6.4: Region; Local and International Organizations

			Organizations		Total
			Local	International	
Region	ERBIL & DUHOK	Count	23	4	27
		% within Region	85.2%	14.8%	100.0%
		% within Local, International	60.5%	80.0%	62.8%
		% of Total	53.5%	9.3%	62.8%
	SULAIMANI	Count	15	1	16
		% within Region	93.8%	6.3%	100.0%
		% within Local, International	39.5%	20.0%	37.2%
		% of Total	34.9%	2.3%	37.2%
Total		Count	38	5	43
		% within Region	88.4%	11.6%	100.0%
		% within Local, International	100.0%	100.0%	100.0%
		% of Total	88.4%	11.6%	100.0%

In Table 6.2, for both regions, the local and international organizations are illustrated. It is shown that the local organizations comprise 38 (88.4%) of which 23 (85.2%) from Erbil and Duhok region and 15 (93.8%) from Sulaimani while international organizations comprise only 5 (11.6%) of which 4 (14.8%) from Erbil and Duhok, and only 1 (6.3%) from Sulaimani region. Moreover, as a result of this survey, it is revealed that, as the region is in its infancy, Data Protection Act (DPA) has not been yet legislated; although there is a common understanding that the main components of this act are followed. In addition, most of the organizations within construction sectors are independent organizations.

Table 6.5: Subsidiary Companies

Subsidiary	Frequency	Percent
Yes	10	23.3
NO	33	76.7
Total	43	100.0

Table 6.3 illustrates that: only 10 (23.3%) of all organizations are subsidiaries for another parent company and the rest, 33(76.7%) are independent organizations. Furthermore, from the analysis of contractor from the analysis of contractor evaluation and contractor selection, it is shown that all four main procedures, pre-qualifications, direct invitation, by negotiation and restricted selection are practiced by both private and public sectors. Following tables, Table 6.4-6.7, explains the results for this analysis.

Table 6.6: Pre-qualification Procedure based on Lower Price

Pre-qualification				based on Lowest Price		Total
				YES	NO	
YES	Type Organization	PUBLIC	Count	**13**	**10**	**23**
			% within Type organization	56.5%	43.5%	100.0%
			% within based on lowest price	72.2%	52.6%	62.2%
			% of Total	35.1%	27.0%	62.2%
		PRIVATE	Count	**5**	**9**	**14**
			% within Type organization	35.7%	64.3%	100.0%
			% within based on lowest price	27.8%	47.4%	37.8%
			% of Total	13.5%	24.3%	37.8%
	Total		Count	**18**	**19**	**37**
			% within Type organization	48.6%	51.4%	100.0%

Table 6.4 explains that, out of 43 participants only 37 (86%) practice pre-qualification procedure, in which 23 (62.16%) are from public sector with 13 (56.6%) based on the principle of lowest price and 10 (43.5%) based on various criteria for their selection process, according to their importance, tender price; time; tender classification; past experience; past performance; financial capability; well-organized tender form; and other. Private sector comprises 14 (37.83%) of which 5 (35.7%) based on lowest price and 9 (64.3%) based on other criteria, mentioned before.

Table 6.7: Direct Selection (Invitation) Procedure

DIRECT Selection Procedure				PROJECTS ARE VERY SPECIFIC?		Total
				YES	NO	
YES	Type Organization	**PUBLIC**	**Count**	**14**	**5**	**19**
			% within Type organization	73.7%	26.3%	100.0%
			% within Projects are very specific?	73.7%	50.0%	65.5%
			% of Total	48.3%	17.2%	65.5%
		PRIVATE	**Count**	**5**	**5**	**10**
			% within Type organization	50.0%	50.0%	100.0%
			% within Projects are very specific?	26.3%	50.0%	34.5%
			% of Total	17.2%	17.2%	34.5%
	Total		**Count**	**19**	**10**	**29**
			% within Type organization	65.5%	34.5%	100.0%
			% within Projects are very specific?	100.0%	100.0%	100.0%
			% of Total	65.5%	34.5%	100.0%

Hence, from Table 6.5, number of participants who practice direct selection procedures is 29 (67.44) of which 19 (65.51%) are from public sector where 14 (73.7%) practice it for very specific projects while 5 (26.3%) practice it based on different criteria according to their importance, security reasons; resources (human and technical); financial capability; past relationship; past performance; continuation for previous project; tender price and other. For private sector, 10 (34.48%) practice this procedure with 5 (50%) use it for very specific projects and other 5 (50%) is based on the criteria mentioned before.

Table 6.8: By Negotiation Procedure

BY NEGOTIATION Procedure				PROJECTS ARE VERY SPECIFIC?		Total
				YES	NO	
YES	Type Organization	**PUBLIC**	**Count**	**12**	**5**	**17**
			% within Type organization	70.6%	29.4%	100.0%
			% within Projects are very specific?	70.6%	45.5%	60.7%
			% of Total	42.9%	17.9%	60.7%
		PRIVATE	**Count**	**5**	**6**	**11**
			% within Type organization	45.5%	54.5%	100.0%
			% within Projects are very specific?	29.4%	54.5%	39.3%
			% of Total	17.9%	21.4%	39.3%
	Total		**Count**	**17**	**11**	**28**

In Table 6.6, it is shown that only 28 (65.11%) practice by negotiation procedure of which 17 (60.7%) are public organizations, 12 (70.6%) of them use this procedure for very specific projects and other 5 (29.4%) depend on various criteria which are, according to their importance, security reasons; special requirements; tender price; resources (human and technical); financial capability; past relationship; past performance; continuation for previous project and other. 11 (39.3%) are from private organizations with 5 (45.5%) practice it for very specific projects while 6 (54.5%) depend on the criteria mentioned before, for the selection process.

Table 6.9: Restricted Selection Procedure

Restricted Selection			PROJECTS ARE VERY SPECIFIC?		
			YES	**NO**	**Total**
YES	**PUBLIC**	**Count**	**9**	**2**	**11**
		% within Type Organization	81.8%	18.2%	100.0%
		% within PROJECTS ARE VERY SPECIFIC?	50.0%	50.0%	50.0%
		% of Total	40.9%	9.1%	50.0%
	PRIVATE	**Count**	**9**	**2**	**11**
		% within Type Organization	81.8%	18.2%	100.0%
		% within PROJECTS ARE VERY SPECIFIC?	50.0%	50.0%	50.0%
		% of Total	40.9%	9.1%	50.0%
	Total	**Count**	**18**	**4**	**22**
		% within Type Organization	81.8%	18.2%	100.0%
		% within PROJECTS ARE VERY SPECIFIC?	100.0%	100.0%	100.0%
		% of Total	81.8%	18.2%	100.0%

Finally Table 6.7 explains restricted selection procedure; the number of construction organizations that use this procedure is 22 (62.79%), 11 (50%) of them are from public sector with 9 (81.8%) practice it for very specific projects and the other 2 (18.2%) depend on various criteria listed, according to their importance, as security reasons; financial capability; past relationship; past performance; continuation for previous project and other, while only 11 (50%) are from private sector with 9 (81.8%) use it for very specific projects while the rest based on, in their selection, the criteria mentioned before.

Remark 6.3

In KR of Iraq, as two different financial regulations are followed, in this survey, two different regions are considered. For each sector, the public or private, multi-tendering procedures are utilized; each one is based on various criteria that are necessary to compliance with the specific requirements of each project.

6.5.2.1 Analysis of the most significant criteria for a successful selection process

In this section, the analysis of the most significant criteria for selecting the best tender, from a list of measurements, which is obtained from conducting the survey study, that are listed, according to their importance, as lowest tender price; tender's qualification and skill; financial capability; past performance; past experience; resources; current work load; currency stability; material price stability; governmental support and other, is practiced. It ranks these criteria, according to the degree of agreements of the participants, from the most significant criteria to the least one. Then from this analysis, the new ranking, for the most significant criteria, based on the judgments of experts in KR, is identified and the weights for each of these criteria are determined.

From the empirical assessment, the data analysis for the five main degrees of agreement, 1-strongly disagree; 2-disagree; 3-neither agree nor disagree; 4-agree and 5-strongly agree, to each of the main criteria, listed above, is obtained based on the consensus of agreements. This analysis gives both past experience and past performance the same importance as both have the same degrees of agreement which is between strongly agree and agree. However, the past experience criterion is considered to be of higher rank than of past performance because the former takes a longer period to give high rank experience. While the consensus of agreement, of participants to this survey, as an example, for current work load or for lower tender price is mostly around disagree; hence they are with lowest ranking order. Thus, the data analysis for this survey conducts a new criteria ranking, which is listed below

1. Past experience;
2. Past performance;
3. Tender's qualification and skill;
4. Financial capability;
5. Governmental support;
6. Material price stability;
7. Resources (human and technical);
8. Currency stability;
9. Current work load;
10. Lowest tender price;
11. Other.

Thus, the most important criterion, Past experience, is given the rank order $r_1 = 1$, and the second most important, Past performance, is given $r_2 = 2$ and so forth; the method of rank sum weighting maps the ranks linearly (i.e. with equal distances) on the interval [0:1] (Schutz, et al, 2006), and the weight for each criterion can then be determined using the following formula:

$$w_i = \frac{(n - r_i + 1)}{\sum_{k=1}^{n}(n - r_k + 1)} \tag{6.1}$$

Where n is the total number of criteria, the sum of weights equal 1.

From (6.1), the weight $w_i, 0 \le w_i \le 1, i = 1,11$ for each of these criteria (from past experience to the lowest tender price and other) from the highest to the lowest can easily be determined:

$$\begin{aligned} &w_1 = 11/66 = 0.1667\ ;\ w_2 = 10/66 = 0.1515\ ;\ w_3 = 9/66 = 0.1364 \\ &w_4 = 8/66 = 0.1212\ ;\ w_5 = 7/66 = 0.1061\ ;\ w_6 = 6/66 = 0.0909 \\ &w_7 = 5/66 = 0.0758\ ;\ w_8 = 4/66 = 0.0606\ ;\ w_9 = 3/66 = 0.0455 \end{aligned} \tag{6.2}$$

$w_{10} = 2/66 = 0.0303$; $w_{11} = 1/66 = 0.0152$, where $(1+2+3+,...+11) = 66$, w_1 represents the first criterion (past experience), w_2 represents past performance, .., w_{11} represents the last criterion, other.

6.5.2.2 Analysis of key factors to failure of a project

In this analysis, empirically, the reasons for the failure of the implementation of the agreed tender, according to the degree of agreements, are analyzed and a new ranking order for the list of critical reasons, which cause delivery problems, is obtained. According to their importance, the following list shows the new ranking of the significant criteria, according to their importance from the highest to the lowest

1. Unqualified tender and his staff;
2. No quality control for materials;
3. Lowest tender price;
4. Unsettled prices for materials;
5. Transferring tenders to another part;
6. Tender's work load;
7. Lack of governmental support;
8. Political situation;
9. Other.

In this analysis, for example, the degree of the consensus of agreement for unqualified tender and his staff is between 4 and 5 (strongly agree and agree), hence it is with higher rank; while this degree for political situation is around 2 (disagree) then it is with the lowest rank or weight.

Similarly, from (6.1), the weight for each criterion is determined.

$$w_1 = 9/45 = 0.200 \;;\; w_2 = 8/45 = 0.177 \;;\; w_3 = 7/45 = 0.155$$
$$w_4 = 6/45 = 0.133 \;; w_5 = 5/45 = 0.111 \;; w_6 = 4/45 = 0.088 \tag{6.3}$$
$$w_7 = 3/45 = 066 \;;\; w_8 = 2/45 = 0.044 \;;\; w_9 = 1/45 = 0.022 \text{ , where } (1+2+3+,...+9) = 45$$

Remark 6.4

From the data analysis for the survey study, it is deduced that, for identifying the most qualified tender, various criteria, which are found to be with higher ranks, such as past performance, past experience for the client in addition to his qualifications and skill with the financial capability and other criteria, are put forward by the respondents to be considered as significant criteria to reach at a successful selection process. Furthermore, despite that the principle of lowest tender price has been practiced, it is suggested that it should not be considered as the main criteria; majority of the respondents think that it is a key factor, in addition to other criteria such as unqualified tender; fluctuation of material prices and others, for the failure of a project. This supports findings from other studies, which conclude that practicing lowest tender price in the selection process cause project failure, (Samuelson and Levitt, 1982; Russel et al., 1990; Crowley and Hancher, 1995; Herbsman, 1995; Mahdi et al., 2002; Wong et al., 2002; Banaitiene and Banaitis, 2006; Halil, 2007; Laryea and Hughes, 2008).

6.6 Model Implementation

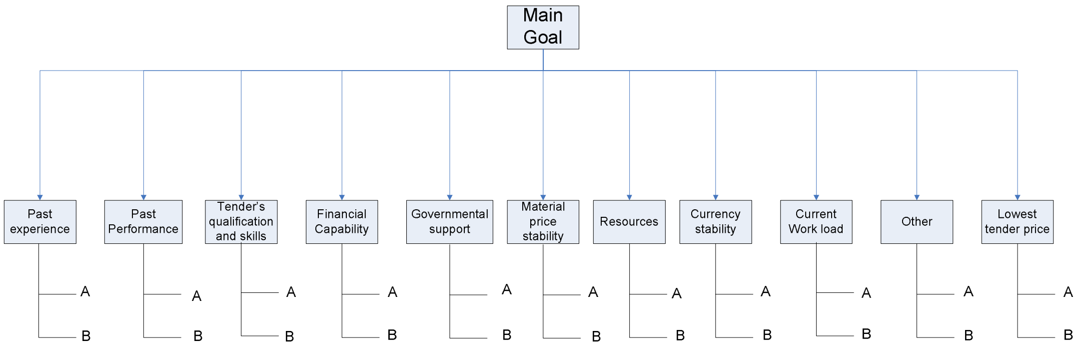
In this section, in order to follow an inductive approach to construct best tender selection strategies in KR, it is attempted to link the theoretically developed approach, which was explained earlier, with the deductively constructed criteria, which are obtained as the result of conducting the main survey, for the same purpose; it is to apply the proposed AHP-based model as a structure procedure, to the field of construction investments, the TSP in KR of Iraq. It is assumed that a scenario where a pair of alternatives, each represents a construction tender for a construction-project, faces with a decision-making problem, then the selection of the most qualified tender based not only on the lowest price but also other specific criteria, extracted from the main survey, is the main objective. The data, which was resulted from the main survey, for identifying specific criteria that were believed to have great influence on the process of selecting the best tender, are used.

Moreover, for the implementation of the proposed model, a team of experts is needed to set up preference matrices, representing expert's judgments. In this region, due to the lack of experience and rarity or insufficiency of experts in this field of the study, it is often hard to find respondents who have knowledge and experience to participate. Therefore, in this study, in order to overcome this difficulty, a new algorithm is proposed. Using simulation, the proposed algorithm generates preference matrices to represent real judgmental matrices, each with a specified level of uncertainty; it is explained later in this chapter. Hence, a single overall score indicator, for each alternative option, is determined, and the most qualified option is that one with the highest score. In order to explain the applicability of the proposed model to the field of construction investment, the main structure of the model is illustrated by the following hierarchy.

6.6.1 Hierarchic structure for the AHP-based model

In this hierarchy, the overall objective, the main criteria, and alternative options are identified. The overall objective, the main goal of this study, is the selection of the most qualified tender located on the top/first level of the hierarchy. The main 11 criteria, Past experience; Past performance; Tender's qualification and skill; Financial capability; Governmental support; Material price stability; Resources (human and technical); Currency stability; Current work load; Lowest tender price and Other, are included in the second level with a pair of alternative options, each representing a construction tender in the final level. The hierarchical outline for this scenario is shown by Figure 6.9.

Figure 6.7: Hierarchic Structure



6.6.2 Methodology for the proposed model

In the selection process, for the implementation of the developed model, based on an AHP that includes various criteria, quantitative measures in addition to qualitative ones, 11 most significant criteria, resulted out from the main survey, explained earlier in this chapter, are considered; each with an estimated weight, $w_i, i = 1,..,11$ determined from (6.1). These criteria are: Past experience, Past performance, Tender's qualification and skill, Financial capability, Governmental support, Material price stability, Resources (human and technical), Currency stability, Current work load, Lowest tender price and Other; they are located in the second level of the hierarchy and a pair of alternatives (tenders) A, B in the third level . Finally the first level is defined as the main goal which is the selection of the most qualified tender based on these criteria, listed before. Hence, elements from third level are compared pair-wise, with respect to their importance; to each element at the second level and then local priorities for each alternative is determined using Eigen-vector formula. In this application, however, even in the general case, there is no grantee that, it is with no error; the precise values of the pair-wise ratios of the weights can only be estimated. In this application, it is assumed that the relative weight for each criterion is estimated from the determined values $w_i, i = 1,..,11$, and hence, there is no need for a pair-wise comparison of the second level with respect to the first one. Then the weight for each of the alternative option is aggregated and then considered as the total weight resulted from applying the AHP that will be multiplied by the expected utility determined previously to give the final weight, which is considered as an overall score indicator, for each alternative option. Then, ranking process is based on the weight indicators determined from applying this approximation approach.

6.6.3 Pair-wise comparison and determination of weight scores

For the proposed AHP-based model, the general procedure for conducting pair-wise comparison matrices is to invite a team of experts to give subjective judgments to compare elements of each level with respect to an element in a level higher to it, using a 9-point measurement scale. However, in KR, despite the rarity of experts, knowledgeable experts, even if exist, are few and located in different areas or they might not be aware of the importance of their judgments, therefore, might be unfair in their preferences.

Therefore, for the implementation of the proposed model to this case study, in order to overcome the difficulty of finding a group of experts who can participate in setting up the preference matrices, a computer-based algorithm, is proposed. Using simulation, this algorithm, instead of considering expert's judgments, generates random numbers to elicit judgments. Thus, for setting up each comparison matrix the algorithm generates, randomly, point scales between 1/9-9; each one refers to a degree of importance of the two compared elements or their reciprocal values, explained in Table 2.1; as pairs of options are considered then each of these matrices is $2x2$ matrix, hence each is a consistent matrix (Table 2.2).

The generated matrix is then converted in to a normal reciprocal matrix, using Algorithm 5.1; where each reciprocal matrix, with its main diagonal elements equal 1, has its uncertainty bounded to a certain level identified by the user of this algorithm. The relative weight for each of such matrix is then estimated, using the Eigen-value method, to give the local priorities of that pair with respect to specific criteria. This procedure is repeated for all 11 criteria, and local priorities are determined. For an easy computation the software Expert

Choice is used; which makes calculations less complexity, time consuming and therefore easy to use.

For the second level, where the comparison matrix represents the comparison of all criteria with respect to the main goal, it is assumed that the local priorities, weights, have been estimated, from the determined weights, from (6.2), and hence, the obtained measures, weights for these criteria, are considered as the Eigen-values for the comparison matrices; this helps reduce the number of comparison matrices and then simplify the final computations. Then, the overall relative weights for various levels are aggregated to obtain the aggregated weight for each alternative option. Finally, in order to obtain a single overall performance indicator for each option, indicates the final weight scale, the expected utility weight and the AHP weight score, from both approaches, the preference ranking model based on MVA and AHP-based model, are integrated in a multiplication process; the aggregated weight is determined from the multiplication of the two measure scores. This final weight is used as an efficient tool for ranking a pair of alternative options; the one with higher weight is preferred more.

The procedure of generating preference matrices for conducting pair-wise comparisons and then implementing the AHP-based model with specified level of uncertainty is explained.

6.6.4 Algorithm 6.1

The stepwise algorithm for implementing the new model is described:

Step 1: Define the decision problem and the main goal.

Step 2: From the main survey, collect all possible information about the main criteria for the decision problem.

Step 3: Structure the hierarchy from the top through the intermediate to the lowest level.

Step 4: Construct, for the third level, 11 preference matrices, with one matrix with respect to each element in the second level, using steps 4.1 to 4.4;

Step 4.1: Generate, randomly, a preference matrix, for setting up judgments, to carry out pair-wise comparison using a nine-point scale, 1-9, or their reciprocal values; these scales are given by Table 2.2,

Step 4.2: Construct, for the generated matrix, a reciprocal normal random matrix to represent such matrix, using Algorithm 5.1. This new reciprocal matrix is with a specified level of uncertainty σ. The value of σ is identified, by the decision maker, at the beginning of the process.

Step 4.3: Estimate the relative weight, importance, for the decision factor using the Eigen value method; Expert Choice software can be used to calculate the Eigen-vectors.

Step 5: Assume that the determined weights for main criteria, from (6.1), are the estimated weights for criteria in the second level

Step 6: Aggregate relative weights of various levels, obtained from Step 4.3, to obtain the ranking scale for the decision alternatives.

Step 7: Compute, for each alternative, the total measure scale, which is used for ranking, from the multiplication of the relative weight obtained from Step 6 by its corresponding one determined from the expected utility, computed earlier.

Step 8: Rank the two alternatives, based on the computed measure scale from Step 7, to identify the most qualified option.

In this algorithm, when the decision problem and the main goal are defined, then the hierarchy, from the top through the intermediate to the lowest level, for the main problem is structured. The algorithm, using 9-point scale, generates 11 different preference matrices each represents the comparison matrix of the two alternative options with respect to one of the 11 criteria. For each of these generated matrices, using Algorithm 5.1, a random normal reciprocal matrix is generated, and then from this converted matrix an Eigen-vector is determined, using Eigen-value method. For the second level with respect to the first one, it is assumed that the estimated weights for all criteria are determined, from (6.2), then relative weights for the two levels are aggregated to set up the weight measure scale for each alternative options. Finally, the integrated weight, from the multiplication process of two weight scales are used for ranking such pair. This measurement algorithm organizes the proposed evaluation process and helps make it less complex, time consuming and therefore easy to use.

Remark 6.5

In KR, due to the rarity of experts and professionals who are aware of the importance of their judgments to the decision process, Algorithm 6.2 helps in setting up the necessary judgmental matrices that compliance with the specific conditions for the proposed model. Therefore, this algorithm introduces a novel approximation tool that can help in replacing the absence of a team of experts.

Therefore, in order to illustrate the applicability of this model for financial scenarios, one examples is considered; it was applied to ranking preference models earlier; the weighted score resulted out from applying this model is needed to be integrated with that one determined from applying the AHP model to estimate the final ranking scores.

Example 6.1

An example, which was considered previously and implemented by Chapter 4 and the results of implementation was given by Table 4.1, for determining the expected utility for a pair of random variables, is considered. For this chapter, each alternative is assumed to represent a construction tender; the selection process is based on a number of criteria each with estimated weight extracted from the main survey and the aim is the selection of the best option.

In this example, a pair of random variables (X, Y), representing financial investment options, construction tenders, is considered; it is normally distributed with different means and the same variance, i.e. $X \sim N(1,1)$, $Y \sim N(2,1)$, each with the an estimated expected utility, which is determined earlier and explained by Table 4.1. Hence, to implement the developed model as a structured procedure to select, for such pair, the most qualified tender, Algorithm 6.1 is applied. It generates preference matrices with specified level of uncertainty, which is given by the average variation of mapping judgments with the nine-point scale; it is assumed to be equal to 0.5. This means that each generated preference matrix has a level of uncertainty not exceeds the value 0.5. Then the steps of the algorithm are processed. The main preference matrices for the implementation of this example are explained by the following tables, Tables 6.8.

Tables 6.10: Preference Matrices

1	X	Y	Eigen Vector
X	1	1/5	0.3248
Y	5	1	0.6752

2	X	Y	Eigen Vector
X	1	1/3	0.2511
Y	3	1	0.7489

3	X	Y	Eigen Vector
X	1	5	0.7997
Y	1/5	1	0.2003

4	X	Y	Eigen Vector
X	1	1/7	0.1378
Y	7	1	0.8622

5	X	Y	Eigen Vector
X	1	1/3	0.7581
Y	1/3	1	0.2419

6	X	Y	Eigen Vector
X	1	1/9	0.1071
Y	9	1	0.8929

7	X	Y	Eigen Vector
X	1	1/3	0.6715
Y	3	1	0.3285

8	X	Y	Eigen Vector
X	1	1/5	0.1764
Y	5	1	0.8236

9	X	Y	Eigen Vector
X	1	1/5	0.1835
Y	5	1	0.8165

10	X	Y	Eigen Vector
X	1	3	0.675
Y	1/3	1	0.3249

11	X	Y	Eigen Vector
X	1	1/5	0.3248
Y	5	1	0.6752

In Tables 6.8, each one represents a preference matrix, obtained from applying Algorithm 6.1, with respect to one of the 11 criteria. For example, in the first table, 1 refers to the first criteria, Past Experience; 2 in the second table refer to the second criteria, Past Performance; and so on. In each table, the final column represents the Eigen-vector for the final normalized preference matrix; the first value, for this column, indicates the estimated weight for the first option and the second value indicates the one for the second option. The estimated weights for all criteria, in the second level of the hierarchy, are assumed to be weights extracted from the judgments of experts from the main survey; it is given as w_i where

$$w_i = 0.1667; 0.1515; 0.1364; 0.1212; 0.1061; 0.0909; 0.0758; 0.0606; 0.0455; 0.0303; 0.0152$$

Then aggregating all weights, the final approximated weight for each of the two alternative options is determined. Hence, estimation for each weight is given:

$$\text{For } X = 0.40\,; Y = 0.59 \tag{6.4}$$

From (6.4), it implies that the weight for alternative Y is greater than that for X; therefore, it has a higher rank. Thus, option Y is preferred more.

While the obtained results from implementing preference ranking models to the same example, previous chapters, have shown that the expected utility, for the two alternative options, are:
$E[U(X)] = 0.748; E[U(Y)] = 0.99$

Thus, an approximation for the estimated weight scales for this pair, which are obtained from preference ranking model, can be assumed as:

$$X = 0.74;\ Y = 0.99 \tag{6.5}$$

Hence, applying the new integrated AHP-model to this example gives the final scores; they are approximated as:

$$X = 0.4 * 0.74 = 0.3\,;\ \ Y = 0.99 * 0.59 = 0.6 \tag{6.6}$$

However, for the three results, equations (6.4); (6.5); (6.6), the decisions are almost the same; the second option, which is a construction tender that is normally distributed with

(2,1), is assumed to be preferred to the first one, which is normally distributed with (1,1); the strength of preferences are different. The obtained results show that: with the integrated formula from equation (6.6), Y is twice as important as X ; while with others the strength of preferences are much less. Thus, the integrated model gives a more specific indicator that can be relied on for the final decision.

Remark 6.6

TSP has been considered as a significant issue for the implementation of successful projects in the field of financial investments over the past few decades. The definition of success is both objective and subjective, it varies according to clients and experts based on different criteria. Despite that various approaches, based on AHP, has been proposed to offer comprehensive solution for the systematic evaluation for TSP; it has been applied to real-life case studies for various countries , for example, Hatush and Skitmore (1998), UK; Fong and Choi (2000), Hong Kong; Mahdi, et al. (2002), Kuwait; Banaitiene and Banaitis (2006), Lithuania; Bertolini et al (2006), Italy; Halil (2007), Malysia, none has been introduced for the same purpose in KR. However, the implementation of the proposed AHP-preference model based on specific criteria, which are identified by experts in the region, that are constructed for the same purpose show that this integrated model, based on more than one approach, is an efficient approximation tool that can be used as a systematic procedure for TSP for identifying the most qualified tender. It has the capacity to handle a great number of criteria in addition to the qualitative criteria to represent the subjective perception of qualified experts; the model minimizes the required pair-wise comparisons, which is considered to be a major default of AHP. Moreover, it is as an effective tool for formulization of knowledge, which may be considered as one of the main contribution to this study.

Finally, in order to guarantee the sensitivity analysis for the outcomes of the problem or to determined the impact that the actual outcome of a particular variable will have if it differs from what was previously assumed, the subjective judgments of experts in each row were modified slightly to indicate small changes in the judgments, for example, a change from strongly to very strongly and another change from strongly to mildly strong were practiced. For almost all cases, the obtained results indicate that the evaluation of priorities are insensitive to the minor changes occurred in the judgments.

6.7 Summary

The main aim of this chapter was to demonstrate the applicability of the proposed approaches through the implementation of the developed models and algorithms to a field of construction industry, the process of tender selection in KR. Through an investigation to this process, it reviewed the criteria employed in the selection of the most qualified contractors and evaluation of tenders. In addition, the improvement of the system of evaluation for selection of a contractor not only accords to the bid price but also according to other quantitative and qualitative criteria. Tender evaluation has long emphasized that lowest tender price is the most significant criterion in the selection of a best contractor while other quantitative and qualitative evaluations for contractor criteria has less attention. Findings from this investigation, as for other studies investigating other countries, explained that clients want the best possible selection therefore; tenders should not be selected according to the lowest price, but according to the highest weight determined from other qualitative criteria, such as past experience and past performance for the contractors before considering bid price.

Moreover, the survey evidences also revealed the influence of other significant criteria, the qualification of the tender and his staff; financial capability; resources and others on the selection process. Other criteria such as governmental support for construction organizations, private sector, should be considered in providing secure environment for implementation of the tenders and the possibility of helping in making the materials prices settled. Hence, based on these criteria, an AHP evaluation method may be used for the evaluation of the contractor bids. Most of these evaluation methods, however, need a group of respondents and experts to give their subjective judgments arranged as preference matrices, the proposed AHP-model in this study enabled the decision maker to generate, using simulation methods, preference matrices to represent real preference judgments, in case of the absence of judgments. Meantime, instead of considering pair-wise comparisons within AHP for the second level, this approach considered the estimated weights for the main criteria, extracted from this investigation, as the priority weights for these criteria with respect the main goal.

Hence, the overall weight for each option is determined from the multiplication of two weight scales; first is the expected utility determined from the preference ranking models, which are based on MVA consistent with EUT, and the second is the one obtained from using the modified AHP. Then, ranking is made based on the final weight scores determined from the multiplication of two different weights, hence if one of these weights is insufficient for ranking the other one can compensate.

In this chapter, it is concluded that the approximation approach in this study is applicable; it is flexible and appropriate for the process of tender selection, which enables decision makers examine the strengths and weakness of the options by comparing them in pairs with respect to most significant criteria available. Furthermore, in order to control the level of uncertainty inherent with AHP models, an algorithm generated matrices with a level of uncertainty, represents average variance for the Eigen-vector, which does not exceeds a certain limit.

Therefore, the developed AHP-based model helps the decision maker manage a set of specific criteria with a pair of alternatives, each represent a financial scenario in the field of construction specifically tender projects, then selects the most qualified tender. Using simulation, it generates data and creates a feedback system that can be practiced for the evaluation of future projects in addition to its capability to make data handling easier and the evaluation process less complex and time consuming. Therefore, it is considered as an effective tool for formulization of knowledge, which may be regarded as one of the main contribution to this study.

Finally, from the results of the case study, the simulated results for Example 6.1, it can be concluded that the proposed AHP-based model is applicable as a systematic procedure for evaluating tenders, in KR, and can be used as a basis for tender selection process that helps decision makers in reducing time consuming pair-wise comparison judgments. If any new significant criterion emerged to be important that satisfies business needs, it can easily be included in the model. Furthermore, for the evaluation team of experts, a group of knowledgeable people in the area of the study, new members can be included at any time; there is no certain limit for the number of experts, it depends on the availability of their existence and the extent to which they are prepared to be in the evaluation team. However, in any circumstances, if there is difficulty in finding a group of experts, using simulation, generating preference matrices are always possible. Finally, the proposed model in this study is found to be an effective tool for formulization of knowledge, which may be considered as one of the main contribution to this study

CHAPTER SEVEN: CONCLUSIONS AND FUTURE WORK

Technical systems are increasingly designed and optimized, with the help of computer programs, to solve decision making problems in various fields of applications. These programs simulate the dynamic behaviour of systems based on mathematical models. In this research study, based on cumulative functions, using simulation, a novel preference methodology for ranking pairs of uncertain lotteries/random variables with non-negative outcomes was introduced. It was aimed at aiding decision makers in selecting, with the available information, the best decisions possible for scenarios that concern financial economics. This chapter summarises the main undertaken parts of the research in this thesis with the achievements accomplished and the main contributions with their significances. Hence, to expand the research, some possible future research lines are proposed.

7.1 Achievements and Conclusions

In this thesis, a new methodology to modify Mean Variance Analysis (MVA), which is consistent with Expected Utility Theory (EUT), to be based on cumulative function was introduced. Using simulation, a novel preference strategy for ranking pairs of uncertain lotteries/random variables with non-negative outcomes, each represented a financial option, was employed. In this approach, instead of dealing with probability distribution function cumulative functions were considered to propose new models that succeeded in dealing with the deficiency of the existing model, Mean Variance Model (MVM), and provided solutions for large-scale decision problems that concern financial scenarios. Simulation was used to generate, from the inverses of cumulative functions, pairs of random variables; each one represented a pair of decision options/lotteries. The generated pairs, based on the determined values of a function that is defined over the first two moments the mean and the variance, were used for eliciting preferences. For the implementation of the proposed model, it was applied to real-life scenarios, financial options, where investors assumed to make their decisions among pairs of such options; the mean, the variance and the expected utility for each of the generated variable were determined and the preferred option was identified; it was the one with the higher expected utility. Hence, the approximation model proposed under this approach, which was focused on ranking pairs of lotteries/uncertain alternative options, was found to be a flexible preference ranking tool with many desirable properties; it could overcome the shortcomings of MVM when it is applied to financial decision problems. Empirical evidences from simulation results ensured that the new proposed algorithm that described the new approximation model provides a preference basis; it could easily deal with normal random variables with equal means and different variances or difference means and equal variances subjected to many desirable utility functions such as quadratic, exponential and linear plus exponential utility function, with different parameters. This could be compatible with some other models, for example, Bell (1988; 1995); Jia (1995, and support their findings, in addition it resolve their shortcomings, in dealing with different random variables and various utility function.

Furthermore, findings from the implementation of this approach showed that, for pairs of lotteries with non-negative outcomes, based on their variances, ranking preferences are possible while the lottery means are equal, this ensured that regardless of their distributions, eliciting preferences are possible even if only variances are considered. Therefore, this significant result could aid in resolving the deficiency of other approaches, for example, Sarin and Weber (1993), and helped in providing solutions to large scale decision problems that involved risk as an important element as for alternative options representing financial scenarios; this led to propose a risk-ranking strategy, using the same methodology, based on risk measurements.

Hence, when risk is considered as an important component in the decision problem, the study explored a new modification for MVA that is based on variance measures. It introduced a risk-preference model that linked preference ordering of pairs of lotteries, with non-negative outcomes only, directly to a risk ordering, on risk factors. Each risk factor, which is obtained from a multiplication decomposition of such lottery in to its mean multiplied by a risk factor, was defined as the ratio of the lottery relative to its mean and represented by a normalized random variable/lottery with the same expected value. With the existence of EUT, the preference ordering over any pair of such lotteries was converted to a risk ordering on the risk factors obtained from such a decomposition structure. The applications to this framework showed that this approximation model had the potential for resolving deficiencies of the existing model, MVM; it was compatible with other risk-value frameworks, for example, Bell (1988; 1995); Jia (1995), for modelling risk preferences, which are only related to specific utility functions such as power and logarithmic functions. This new methodology, based on simulation, was found to be appropriate for modelling risk preferences; the new approximation approach had the potential to accommodate various distribution functions with different utility functions and usable to handle large-scale decision problems especially those encountered in financial problems. Based on the determined values of variances; the preference ordering could be verified by just focusing on their risk measures.

However, the main shortcoming of such methodology was the way in which the preference information was processed; it was rather objective, which was based on weight scales represented by the determined values for the expected utility; the subjective judgments for the decision makers, which may have great impact on the decision process, was ignored. Therefore, an Analytic Hierarchic process (AHP) model was proposed to allow decision makers and experts include their intuitive perceptions or judgments in the decision process. Meantime, in order to deal with the shortcoming of the AHP, using simulation, an algorithm was proposed to control the uncertainty, which is inherent with the method, and limited it to an accepted level. The main contribution of this algorithm was in modifying AHP to correspond with the problem of uncertainty as the result of mapping judgments into real number. It introduces a novel approximation method that was based on the fact that it was applied to pairs of alternative options, which keeps ranking reversal unchanged; it overcame the limitation of AHP and allowed limiting the uncertainty to a desired level, this can be specified by the decision maker at the beginning of the decision process. The practical consequence of this approach, especially for researchers using AHP as a tool for pair-wise comparisons, is the specification of a boundary for the accepted uncertainty within preference matrices in decision hierarchies. In contrast, other approaches, for example Zahedi (1986); Paulson and Zahir (1993); Zimmermann (2000); Millet and Wedley (2002); Wu (2007); Shibatal, et al. (2009) mostly deal with uncertainty to analyse its effect on rank reversal but not on specifying its level.

Hence, the potential and the applicability of the designed algorithm were ensured; it was applied to a real-life example. Later in this research study, an integrated approach that combined the two modified approaches, namely, MVM and AHP, was established. An algorithm, which describes the steps of integrating the two approaches, for conducting the new approximation model, which is an integrated-AHP model, was introduced. The proposed model included all influencing factors in the decision process and provided an aggregated weight scale, obtained from multiplying two weight scales; the first value was the expected utility determined from applying the modified MVM, and the second was resulted from implementing the modified AHP-based model. From the implementation of this approach to different examples, findings asserted that it was an applicable model for the evaluation of pairs of lotteries representing financial scenarios and the aggregated scale weight was found to be a more accurate weight indicator for the selection process than any other approach when each was implemented independently. Furthermore, the obtained results ensured that this new approximation methodology, based on cumulative functions and simulation results, was an efficient tool to propose an appropriate model to solve decision making problems under uncertainty for scenarios with only non-negative outcomes, financial investment was an example.

However, the proposed model, by this methodology, had the capacity to handle a great number of different criteria in a way that truly reflects the complex reality to incorporate the specific conditions of the project in addition to the user's experience and subjective perception in to the decision process. There was no need to impose any restrictions on the distribution functions for the represented random variables or utility functions, with the help of MAT LAB software, using simulation, enabled the generation of numerous random numbers, from the inverses of various cumulative distribution functions, possible to represent uncertain options. Meantime, sensitivity analysis, which aimed at examining how changes of judgmental values might affect the ranking results of the decision problems, was developed. It indicated that the evaluation of priorities are insensitive to the minor changes occurred in the judgments.

Finally, this study ended up with the main objective; the demonstration of the proposed approaches through the implementation of the developed models and algorithms in a field of financial economics, construction industry is an example, to solve a real life problem. Hence, the integrated model was used as a structured procedure and applied to a specific case that has significant impact on the success of projects implementation, the Tender Selection Process (TSP) in Kurdistan Region (KR) of Iraq. For this purpose, the study explored the first empirical case study on TSP in KR; it was an inductive and a comprehensive investigation on TSP that has received a minimum consideration in the region; it is considered as a significant contribution to this research. In this study, various construction companies, their type, size, classification and other information was investigated through conducting a pilot study; a questionnaire was constructed based on interviews with a representative sample of construction experts. The designed questionnaire was content validated and completed by a sample of construction experts in KR. Hence, the main survey, based on the obtained information, was conducted; the final questionnaire included detailed information on the construction organizations and their experiences on tenders. Results of the survey were used to identify main criteria, which are believed to have significant impact on TSP with the evaluation of their weights, then to verify the reasons that might cause the delivery problems for the implementation of the entire projects.

The main finding, from the data analysis for the conducted study, was the identification of the analogous criteria for the best selection process in KR as for other countries, for example, UK; USA; Australia; Canada; Saudi Arabia; Singapore; Lithuania; Malaysia, but with different ranking orders. Furthermore, in this region, due to the distinct circumstances, in addition to the practice of the lowest price, the main reason that leads to the failure of the entire project is incomparable to those for these countries; due to high volatility, the currency fluctuation is believed to be the main reason that fluctuate material prices; this directly influenced the implementation of the construction projects.

Hence, the theoretically developed model, an integrated AHP-based model, was linked with the deductively constructed criteria to be used as a systematic procedure for TSP in KR. However, for the implementation of the proposed model to this case study, in order to overcome the difficulty of finding a team of experts who can participate in setting up the preference matrices in Kurdistan Region, a new algorithm was proposed. Using simulation, the proposed procedure generates preference matrices to represent real judgmental matrices, each with a specified level of uncertainty. Thus, for setting up each comparison matrix the algorithm generates, randomly, point scales between 1/9-9; each one refers to a degree of importance of the two compared elements or their reciprocal values. Therefore, this algorithm introduced a novel approximation tool that could help in replacing the absence of a team of experts. Hence, a single overall score indicator, for each alternative option, was determined, and the most qualified option was that one with the highest score.

Therefore, throughout this research study, it can be concluded that the new preference ranking methodology, based on both modified approaches Mean Variance Analysis and Analytic Hierarchy process relying on cumulative function using simulation, is an efficient methodology to propose new models that can rank pairs of lotteries with non-negative outcomes, to solve large-scale decision problems especially those encountered with financial decisions. Specifically, the proposed AHP-based model is found to be applicable as a systematic procedure for evaluating tenders and can be used as a basis for the process of tender selection in Kurdistan Region; it enables decision makers examine the strengths and weakness of the options by comparing them in pairs with respect to most significant criteria available, each with a pre-determined weight, which was obtained from the main survey. In this case, there is no need to do pair wise comparison for criteria with respect to the main goal, this reduces the number of comparison within this model, in addition, Expert Choice software can be used to calculate the Eigen-vectors and provide visual representation of overall ranking on a computer screen; this make data handling easier and the evaluation process less complex and time consuming.

Moreover, using simulation, it generates data and creates a feedback system that can be practiced for the evaluation of future projects in addition to its capability to make data handling easier. If any new significant criterion emerged to be important that satisfies business needs, it can easily be included in the model. Furthermore, for the evaluation team of experts, a group of knowledgeable people in the area of the study, new members can be included at any time; there is no certain limit for the number of experts, it depends on the availability of their existence and the extent to which they are prepared to be in the evaluation team. However, in any circumstances, if there is difficulty in finding a group of experts, generating preference matrices are always possible. Finally, the implementation of the proposed AHP-preference model based on specific criteria, which are identified by experts in the region, that are constructed for the same purpose show that this integrated model, which is based on more than one approach, is capable to offer comprehensive solution for the systematic evaluation for tender selection process for Kuristan Region. Furthermore, findings, from the implementation of the proposed model, show that this approach is compatible with many other approaches that provide solutions for evaluating tenders for various countries, for example, Hatush and Skitmore (1998), UK; Fong and Choi (2000), Hong Kong; Mahdi, et al. (2002), Kuwait; Banaitiene and Banaitis (2006), Lithuania; Bertolini et al (2006), Italy; Halil (2007), Malysia. Moreover, this approach has the capacity to handle a great number of qualitative in addition to quantitative criteria to represent the subjective perception of qualified experts; the model, proposed under this approach, minimizes the required pair-wise comparisons, which is considered to be a major default of AHP. Finally, this approach could be an effective tool for formulization of knowledge, which may be considered as one of the main contribution to this study.

Therefore, the most significant contribution of this research study, based on the obtained results, can be summarised as:

- Further development for MVM, which is consistent with EUT, to comply with the shortcoming of the limitation of the model, in which the probability distribution function should only be normal and the utility function is to be quadratic. Based on cumulative function, the modified model conducts a new methodology that proposes a new modelling strategy for ranking preferences for pairs of options. Meantime utilization of simulation results, to generate knowledge and data, help in simplifying the decision process;
- Improvement in AHP to handle the weakness of the uncertainty that results from the difficulty of setting up the right preference matrices to represent decision maker's judgments. In addition, introducing an algorithm to propose a novel approximation tool that can help in replacing the absence of a team of experts to set up preference matrices. On the practical side, the proposed model offers an efficient, convenient tool that guides the decision makers in to a methodical thinking procedure in order to make logical, consistent decisions and provides a facility for all necessary computations;
- Conducting the first comprehensive survey on construction organizations in KR of Iraq. In this region, through an investigation, for both public and private sectors, the main specific criteria, which are believed to have significant impact on the selection of the most qualified tender, are identified. Then, the main reasons that may cause the failure of the implementation of the agreed tenders, with the evaluation of weights for constructed criteria, are verified.
- Establishment of a novel systematic procedure for the process of tender selection in Kurdistan Region of Iraq, based on the data extracted from the main survey, to correspond with the specific criteria identified by the existing professionals.

This thesis, however, introduced new methodologies and algorithms that help in solving decision problems that concern financial economics; the treatment is far from complete. Here, briefly some possibilities for future work and recommendations are given.

7.2 Future Research and Recommendations

In this section, few lines of future research, which may arise from the developed methodologies and modelling procedures proposed through the chapters of this thesis, are briefly discussed. Recommendation for future work is to extend and build on the obtained research work, conducted in this study; a similar methodology to be applied for group ranking process, which can rank alternative options in groups, taking into account all the different factors involved (both tangible and intangible). This can include the following:

In regard with the first part of the modification of MVA to propose a new ranking procedure, based on cumulative distribution function, using simulation, to handle pairs of alternative options, with only non-negative outcomes, can be extended to handle alternative options in groups. For this process, pairs of these options can be ranked at a time. The same modification in proposing risk-preference model can be extended to yield an efficient selection tool that is capable of handling alternative options in groups; each represents a scenario in finance, taking in to account all significant factors that might have impact on the decision problem. It is anticipated that, in both cases, this modelling procedure can be an efficient decision tool to handle group decision making problem if the problem of ranking reversal is resolved. Rank reversal means that ranking between two alternatives might be reversed after some variation occurs to the decision problem, like adding a new alternative, dropping an old one or replacing a non-optimal alternative by a worse one etc. Usually such a rank reversal is undesirable for decision-making problems; the validity of the method could be questioned if it does allow it to happen. However, some recent studies indicate that rank reversals could also happen because of people's rational preference reversal which may be caused by changing their emotional feelings, and perceptions

In regard with the new proposed AHP-based model, it is recommended that this model can be extended to handle not only pairs of alternative options, but in groups. However, in this case, the major drawback of AHP is the uncertainty that can cause rank reversals of alternative options, which can be handled in different ways and for different purposes. At present there exist a considerable number of theories, methods to model uncertainty; two of the most extended methods are the use of: (1) interval judgements, and (2) probability distributions. It is, however, believed that probability theory is sufficient to model all kinds of uncertainty; it is recommended that uncertainty can be handled better through probability distributions. Therefore, a complete probabilistic extension to the AHP method is recommended to provide the decision maker not only with information on the ranking of the alternatives but also the probability that the ranking remains stable even in presence of uncertainty in the judgements. Hence, more research is needed to study probabilistic and statistical approaches in analyzing rank reversal properties of the AHP methodology with uncertain pair-wise comparison judgments. Simulation experiments can be used as an effective and accurate tool for analyzing the stability of the preference rankings under uncertainty.

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APPENDIX: PUBLICATIONS DURING STUDY

1.

Int. J. Operational Research, Vol. 5, No. 3, 2009 311

A preference ranking model based on both mean-variance analysis and cumulative distribution function using simulation

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Abstract: In decision-making problems under uncertainty, mean-variance analysis consistent with expected utility theory plays an important role in analysing preferences for different alternatives. In this paper, a new approach for mean-variance analysis based on cumulative distribution functions is proposed. Using simulation, a new algorithm is developed, which generates pairs of random variables to be representative for each pair of uncertain alternatives. The proposed model is concerned with financial investment for risk-averse investors with non-negative lotteries. Furthermore, the proposed technique in this paper can be applies to different distribution functions for lotteries or utility functions.

Keywords: mean-variance theory; expected utility theory; cumulative distribution function; simulation.

Reference to this paper should be made as follows: Fatah, K.S., Shi, P., Ameen, J.R.M. and Wiltshire, R.J. (2009) 'A preference ranking model based on both mean-variance analysis and cumulative distribution function using simulation', *Int. J. Operational Research*, Vol. 5, No. 3, pp.311–327.

2.

Int. J. Operational Research, Vol. X, No. X, xxxx 1

A risk-preference model for normalised lotteries based on both mean-variance analysis and cumulative distribution functions

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Abstract: In decision-making problems under uncertainty, analysing preferences for pairs of uncertain alternatives (lotteries) based on mean-variance and cumulative distribution functions have been studied and preference ranking models have been proposed. In this paper, a new risk-preference model for ranking pairs of lotteries (random variables), representing risk factors, is proposed. Each random variable is obtained by converting the outcomes of the lottery into its mean multiplied by a relative risk factor. With the existence of an expected utility model, the preference ordering is converted into a risk-preference ordering over their risk factors. The proposed model is an efficient approximation model, based on simulation results, developed for financial investment for risk-averse investors.

Furthermore, unlike the other models, it can be applied to a variety of randomly distributed random variables with different utility functions.

Keywords: Cumulative distributions function; expected utility theory; mean-variance theory; normalised lotteries; simulation.

Reference to this paper should be made as follows: Fatah, K.S., Shi, P., Ameen, J.R.M. and Wiltshire, R. (xxxx) 'A risk-preference model for normalised lotteries based on both mean-variance analysis and cumulative distribution functions', *Int. J. Operational Research*, Vol. x, No. x, pp.xx–xx.

3.

The following paper has been submitted to *Construction Management and Economics*: RCME-MS-09-0955 (under review)

Construction Engineering Tender Selection: A Case Study in Kurdistan Region of Iraq

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August, 2009

Abstract: During the last decade, as part of construction development plan for Kurdistan Region of Iraq, many strategic construction projects have been undertaken. In order to ensure the success of these projects**,** a structured procedure for the selection of an appropriate and qualified tender is highly critical. To evaluate tenders, various significant criteria including that with the lowest price and the subjective judgments of construction experts should be considered. This paper is an inductive investigation on Tender Selection Process that has received a minimum consideration in this region. Various construction companies, their type, size, classification and other information have been investigated and used expert information to identify common criteria that will have a significant impact on the process with the evaluation of weights for alternative scenarios. In addition, the most crucial reasons that cause the delivery problems are identified. This study is part of a larger one attempting to bridge the theoretically developed models for tender selection with the deductively constructed criteria for the same purpose to construct best tender selection strategies in Kurdistan Region.

Keywords: Tender selection process, criteria evaluation, weight evaluation, survey

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